Nature-Inspired Optimization Algorithms

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Nature-Inspired Optimization Algorithms

A Vasuki

CRC Press

Taylor & Francis Group

Boca Raton London New York

CRC Press is an imprint of the
Taylor & Francis Group, an **informa** business
A CHAPMAN & HALL BOOK

First edition published 2020
by CRC Press
6000 Broken Sound Parkway NW, Suite 300, Boca Raton, FL 33487-2742

and by CRC Press
2 Park Square, Milton Park, Abingdon, Oxon, OX14 4RN

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ISBN: 978-0-367-25598-5 (hbk)
ISBN: 978-0-429-28907-1 (ebk)

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Preface

This book is intended as a reference for undergraduate and postgraduate students as well as researchers who are working on complex problems to find optimum solutions. The book can serve as a reference for faculty who are handling classes on the topic of optimization. It gives a general overview of optimization and its applications with simple examples and discusses conventional optimization techniques and their limitations. Fifteen nature-inspired optimization algorithms have been included with a detailed explanation of the techniques.

These optimization algorithms can be applied to any real-life problem, either constrained or unconstrained. It involves choosing values of parameters associated with the problem in order to arrive at the optimum solution. The unconventional optimization algorithms are broadly based on biological evolution and physical and chemical processes. These nature-inspired algorithms have been developed from the study of swarm behavior of animals, birds, and insects. The main advantage is that they are population-based, and hence the search for the optimum solution can be conducted in parallel by multiple agents. This reduces the time taken to arrive at the (best) global optimum solution to the problem. Since the problems could be multimodal with several local optima, the algorithm should be able to distinguish between local and global optimum solutions.

These algorithms can be tested with standard benchmark data sets and classical engineering design problems. The standard problems in computer science such as the traveling salesman, graph coloring, finding the shortest path in a graph, job scheduling, and routing in computer networks are NP-hard. Image processing is another broad area that includes enhancement, segmentation, compression, classification, object recognition, feature selection, clustering, and registration. The application of nature-inspired algorithms to such intractable problems leads to optimum solutions in the shortest possible time. A list of comprehensive problems upon which these algorithms can be tested is also provided.

I sincerely thank everyone who have helped me in completing this book.

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1

Introduction

1.1 Introduction

Optimization is of prime importance in most of our everyday activities and plays a vital role in achieving desired goals. Optimization is arriving at the best set of parameters, maybe by trial and error, so that we achieve the desired objective(s) exactly or at least a good approximation. We optimize parameters to achieve desired objectives without being consciously aware of it many times in our day-to-day activities. Optimization can be applied in diverse fields such as mathematics, physics, chemistry, engineering design, production, telecommunication, networks, computer science, economics, management, etc. The problems range from the very simple to the highly complex ones like designing steel structures, aircraft routes, road connectivity, routing in computer networks, unmanned space missions, autonomous vehicles, manufacturing, automation, robots, and so on. The goal of optimization could be to maximize quality/profit/efficiency or minimize cost/wastage/resources utilized with reduced time, space, and computational complexity. The problems might have constraints that have to be satisfied in finding the optimal solution. The constraints could be limits on length, area, power consumed, cost, weight, or restriction on certain variables to assume integer values (positive or negative), and some variables need to be bounded between minimum and maximum values as specified in the problem. In all optimization problems there will be several parameters that have to be adjusted to get the best possible output. The problems to be solved are generally complex in nature with many conflicting requirements. The solution attained has to accommodate all these various conflicting requirements, and, keeping the objective(s) in focus, the optimization technique has to be designed.

1.2 Fundamentals of Optimization

Optimization is obtaining the optimum values of a set of variables upon which an objective function depends, and that could be either constrained or unconstrained. Any set of values assigned to the variables always produces an output but the optimum set of values produces the optimum output. The objectives of the problem and the constraints, if any, can be formulated in terms of mathematical functions or equations. The mathematical expression representing the goal of optimization is called the objective function. The objective function could be defined for maximization or minimization depending on the problem domain. Figure 1.1 illustrates the general optimization problem with input

FIGURE 1.1

General optimization problem.

variables and constraints that either maximizes or minimizes the objective function. In Figure 1.1 the objective function $f(X)$ depends on the vector of input variables represented by $X = [x_1 \ x_2 \ x_3 \ x_4]$.

When an objective function is to be maximized, it is called a fitness or quality function. When it is to be minimized, it is called a cost or penalty function. For a minimization function, if the minimum value is zero it is called an error function. The sign of the objective function can be complemented (+/-) in order to transform a maximization problem to minimization and *vice versa*. An objective function that is to be maximized can also be minimized by taking the negative of the function.

Some of the characteristics of the objective function are:

- The number of variables in the function is the dimension (d) of the search space.
- Whether the variables assume continuous or discrete values determines whether the function is continuous or discrete.
- If the function is continuous, is it differentiable at all points in the search space?
- Whether the function has one maxima (unimodal) or multiple maxima (multimodal).
- Is the function unconstrained or constrained? If constrained, how many and what are the constraints? Are they equality or inequality constraints?

Mathematically, a function is represented as $f: R^d \rightarrow R$, where the function f belongs to the d -dimensional hyperspace R^d . The domain R^d is the parameter or search space with each $X \in R^d$ being a possible candidate solution to the objective function $f(X)$. The function $f(X)$ maps the search space to the function space R . The problem is to find $X^* \in R^d$, for which $f(X^*) \geq f(X)$, $\forall X \in R^d$. This applies to a maximization problem, and for minimization problems, it has to be $f(X^*) \leq f(X)$, $\forall X \in R^d$. The search space could be either the entire d -dimensional hyperspace or a subset of the hyperspace. When there are constraints associated with the problem, the search space becomes a subset of the d -dimensional hyperspace. The characteristics associated with the constraints of the problem are:

- Number of constraints for the problem
- Whether the constraints are equality or inequality constraints

FIGURE 1.2

One-dimensional function $f(X)$ where $X = [x]$.

Figure 1.2 shows an objective function $f(X)$ where the vector $X = [x]$ is one-dimensional, since the function depends on only one variable x .

The one-dimensional plot of the objective function $f(X)$ shown in the Figure 1.2 has a maximum value at $x = x^*$. This is the global optimum since this is the highest value for the function $f(X)$. The search space is the one-dimensional x -axis where the continuous variable x ranges from zero to a maximum value x_{max} . In general, the search space is $S \rightarrow R^d$. The convergence of the algorithm or the final value attained is the optimum solution and depends to a great extent on the starting point in the search space or the initial value of x . If the negative of the function $f(X)$ is taken, the maximum value becomes the minimum value or the peak becomes a valley. The maximization problem becomes a minimization problem when the global minimum occurs at $X = x^*$.

Extending this concept further, the one-dimensional function $f(X)$ could have multiple maxima or minima, as illustrated in Figure 1.3. The function $f(X)$ shown in Figure 1.3 has multiple peaks and valleys, where $X = [x]$. The peak with the highest value is called the global maximum and the valley with the lowest value is called the global minimum. The other peaks which are smaller than the maximum are called local maxima, and the valleys which have higher values than the minimum are called local minima. The search for the optimum solution for $f(X)$ starts somewhere along the curve and the objective is to find the global maximum or minimum, as the case may be. The problem occurs when the search for the optimum solution gets trapped in local maxima or minima. The optimization algorithm should be such that the function should attain the global optimum value and if it gets trapped in local optimum it should be able to come out of the local optimum point or region in the search space.

FIGURE 1.3

One-dimensional function $f(X)$ having multiple maxima and minima.

In the above problem the function $f(X)$ is dependent on a single variable x ; hence it is single variable optimization. This maximization problem of one variable can be extended to d number of variables, thus making the objective function d -dimensional and the search space becomes a d -dimensional hyperspace. In effect, this becomes multivariable optimization. As another example, consider a function of two variables $f(X) = f(x_1, x_2)$ with only one global minima. The two-dimensional search space of a function of two variables $f(X) = f(x_1, x_2)$ is shown as a contour plot in Figure 1.4. The global minima appears nearly at the center of the search space, as indicated.

In general,

$$f(X) = f(x_1, x_2, \dots, x_d) \quad (1.1)$$

and the search space is $S \rightarrow R^d$. This is single-objective optimization problem since there is only one objective function that is to be maximized or minimized. In contrast to this, if there is more than one objective function to be maximized or minimized then it becomes multi-objective optimization. If there are no constraints attached to the problem it is unconstrained optimization whereas if there are one or more constraints in the problem it is constrained optimization. The constraints of the problem could be equality or inequality constraints. They are mathematically represented as:

$$\begin{aligned} g_i(X) &= 0, \quad i = 1, 2, \dots, P \\ h_j(X) &\geq 0, \quad j = 1, 2, \dots, Q \\ \text{or } h_j(X) &\leq 0, \quad j = 1, 2, \dots, Q \end{aligned} \quad (1.2)$$

where the number of equality constraints is P and the number of inequality constraints is Q . When there are multiple number of objective functions, the problem is multi-objective optimization. This is mathematically represented as:

$$f(X_k) = f(x_{k1}, x_{k2}, \dots, x_{kd}), \quad k = 1, 2, \dots, K \quad (1.3)$$

where K is the number of objective functions that are to be either maximized or minimized. The objective function $f(X)$ maps the search space to the function space, which will

FIGURE 1.4

Two-dimensional search space of a function $f(x_1, x_2)$ with one global minimum.

have a single value for single-objective optimization and multiple values in the case of multi-objective optimization. The parameters or variables can be continuous or discrete and belong to a finite or infinite set. The parameters could be interdependent or they may be independent of each other. The function might have only one maximum point that is the global maximum called a unimodal function, or it could have one global maximum and multiple local maxima called a multimodal function. The example plotted in Figure 1.2 is unimodal whereas the plot shown in Figure 1.3 is multimodal.

1.3 Types of Optimization Problems

Optimization problems usually start with the definition of the problem such as minimization of cost, energy, resources used, or maximization of profit and quality. From this problem statement the objective function is formulated, either as a maximization or minimization function. The next step is to identify the constraints associated with the problem, which could be either equality or inequality constraints. The parameters associated with the objective function and the constraints have to be identified and their boundaries clearly stated. The optimum solution for the problem has to be found by searching, for which the search or solution space needs to be defined. The tentative location of the solution in the search space or the local region where the solution could possibly be found also has to be initially known, because that is the point at which the search for the optimum has to begin. If the information about the local region where the solution is likely to be found is not available, then the search for the optimum has to start from a random location in the search space. If the space is quite large and multidimensional, it is neither practical nor feasible to do an exhaustive search of the solution space. If there are no objectives in the problem but only constraints, then it is a *feasibility* problem. If the objective function and constraints are *separable* in the design variables, it is a *separable* optimization problem. Given any mathematical function of design variables and constraints, if it is possible to separate them in terms of the variables then it is a *separable* problem. Let a function of three design variables be defined as $f(X) = x_1x_2x_3$. Taking logarithm on both sides, $\log [f(X)] = \log(x_1x_2x_3) = \log x_1 + \log x_2 + \log x_3$. Since this is separable in terms of the three variables x_1 , x_2 , and x_3 , it is a *separable* problem.

The optimization problems and techniques are categorized into several classes based on the characteristics of the objective function, the associated variables, and constraints.

Continuous and Discrete Optimization: When the objective function is a continuous function of design variables it is continuous optimization. This continuous function could be differentiable or non-differentiable. If the function is differentiable, the traditional or classical methods of optimization can be applied. $f(x) = 12x^3 + 5x^2 - 2x + 4$ is an example of a continuous function that is differentiable, and the traditional methods of optimization can be applied to find the minima or maxima of the function $f(x)$. If the function is non-differentiable, then other methods have to be explored, or they have to be transformed such that the classical methods can be applied. Such non-differentiable functions are usually not smooth and have sharp discontinuities where the derivatives do not exist. When the objective function is dependent on a set of independent discrete variables it is discrete or combinatorial optimization. Sometimes, the function could be a mix of continuous and discrete variables and it becomes a mixed optimization problem. An example of a combinatorial optimization problem is finding the minimum spanning tree of a connected

FIGURE 1.5

Minimum spanning tree.

undirected graph. The problem is to connect all the nodes of the graph such that the total weight of the edges connecting these nodes is minimum. In addition, there should not be any cycles in the minimum spanning tree. Kruskal's and Prim's algorithms are two greedy algorithms that are normally applied in finding the minimum spanning tree of a graph.

Figure 1.5 shows an example of a connected graph, and its minimum spanning tree is shown highlighted. Initially starting from node *a* of the graph, all the edges connecting this node are examined and the edge with the least weight of 2 is chosen and the node *b* connecting this edge is included in the tree. The edges connected to nodes *a* and *b* that are not yet included in the tree are examined, and the edge with a weight of 3 connecting node *b* to node *e* is included in the tree. This process is repeated until all the nodes in the graph have been covered. The algorithm for obtaining the minimum spanning tree is a greedy algorithm since it chooses the edge with the smallest weight at each step. The final optimized output is the set of edges whose total weight is minimum with all the nodes of the graph being connected and included in the tree. This is an optimization problem with the objective function being the total weight of all the edges in the minimum spanning tree and the constraints being that each node be included only once and no cycles are formed.

Deterministic and Stochastic Optimization: Deterministic optimization algorithms are consistent, and each time the algorithm is run, it produces the same result for the same input values. Stochastic optimization algorithms have some randomness associated with them and might produce different results for each run of the algorithm. An example for the deterministic optimization algorithm is hill climbing and other traditional gradient-based methods such as the simplex method. Genetic algorithm and particle swarm optimization are two famous examples of population-based stochastic optimization techniques. Stochastic optimization algorithms are heuristic or metaheuristic and have some inherent randomness that leads to the optimum or near-optimum solution in finite time.

Constrained and Unconstrained Optimization: When there are no constraints on the problem to be solved it is unconstrained optimization. When there are constraints related to the problem, it is constrained optimization. Let $f(x_1, x_2) = x_1^2 - 10x_1 + 5x_2^2 - 2x_2 + 3$ be the function to be optimized. The problem is to find the values of x_1 and x_2 that will maximize $f(x_1, x_2)$. If there is no restriction on the values that x_1 and x_2 can assume, it is unconstrained optimization. If it is stated that x_1 can only be a positive integer then it is constrained optimization. The constraints could be equality or inequality constraints. The constraint $x_1 \geq 0$ is an inequality constraint whereas $2x_1 + x_2 = 6$ is an equality constraint. Constrained optimization problems are more difficult to solve than unconstrained optimization problems.

Linear and Non-linear Optimization: If the objective function to be optimized is linear then it is linear optimization. If the objective function is non-linear then it is non-linear optimization. An example for a linear objective function is $f(X) = 2x_1 + 4x_2 - 7$, and a non-linear objective function is $f(X) = x_1^2 + 3x_1 - 2x_2^2 + 5$. Non-linear optimization problems are more

difficult to solve than linear optimization problems. Similar to objective functions, the constraints also could be linear or non-linear, for example, $x_1 + x_2 = 6$ is a linear constraint and $x_1^2 + x_2^2 > 100$ is a non-linear constraint.

Single- and Multi-Objective Optimization: When there is only one objective function $f(X)$ to be optimized it is single-objective optimization. When there are multiple objective functions $\{f(X_1), f(X_2), \dots, f(X_K)\}$ to be optimized it is multi-objective optimization. In multi-objective optimization there are multiple (K) objective functions and each one is a function of d number of design variables. In multi-objective optimization, there could be multiple functions with conflicting requirements. There will be no single design vector that will maximize (or minimize) all the objective functions. It might not be possible to satisfy all the constraints to get an optimum value for all the objective functions. It might be necessary to have tradeoffs when there are conflicting requirements between multiple objectives. A typical example with multiple objectives is manufacturing a car. One objective will be minimization of cost, the second will be minimization of fuel consumption, the third will be maximization of efficiency, the fourth will be minimization of resources used in manufacturing, the fifth will be minimization of emission of pollutants, and so on. Algorithms that work for single-objective optimization might not work directly for multi-objective optimization. One method to solve such multi-objective optimization problems is to construct a single-objective function using a weighted combination of the multiple objective functions such as:

$$f(X) = c_1 f(X_1) + c_2 f(X_2) + \dots + c_K f(X_K) \quad (1.4)$$

where $c_1, c_2, \dots, c_K$ are the weighting coefficients.

A better method to take care of these conflicting requirements is that the optimal values of all the objective functions lie on the Pareto Optimal Front. The Pareto Optimal Front theory states that it is not possible to get the maximum value for all the multiple conflicting objectives of a problem simultaneously. What is stated here for a maximization problem applies equally well for a minimization problem. The optimal method to solve this multi-objective problem with conflicting requirements is to ensure that the solutions to the multiple objective functions lie on the Pareto Optimal Front. When the fitness value of one objective function is increased (improved), the fitness value of another objective function will be decreased (degraded); that means it is not possible to get the maximum value for all the objective functions simultaneously with one solution vector. If a solution vector X_i lies on the Pareto Optimal Front, it yields a vector of fitness values for all the objective functions of the problem. Comparing these fitness values, it occurs that if this vector X_i yields maximum fitness for objective function $f(X_i)$ it might not be the case for the function $f(X_j)$. Another solution vector X_j , which is also on the Pareto Optimal Front will produce the maximum value for the objective function $f(X_j)$ but not for $f(X_i)$ or $f(X_i)$. Therefore, any solution vector X for a multi-objective optimization problem should lie on the Pareto Optimal Front for the solutions to be non-dominated or Pareto Optimal.

1.4 Examples of Optimization

These algorithms have one thing in common, that is, they all strive to produce the best solution for a given problem within a limited set of resources. The problem can be a complex

engineering design such as a CNC Machine, a process plant like paper manufacturing, pattern recognition, image classification, or an NP-hard problem (non-deterministic polynomial time) in computer science like the traveling salesman problem (TSP), and the limitations on the resources are the constraints within which the best solution for the problem is to be attained. It can be expressed in terms of mathematical equations that could be solved using different methods suitable for the problem. One of the most important evaluation criteria for these algorithms is the time, space, and computational complexity incurred in arriving at the optimum solution.

Paper Manufacturing

In a process plant like paper manufacturing, many parameters will be involved in producing the paper. The objective function could be a mathematical function that has to be maximized and indicates the quality of the paper produced. There are several processes involved in paper manufacturing such as pulp extraction, the right blend (proportion) of chemical additives, fixing temperature and/or pressure, drying, bleaching, etc. Choosing the appropriate values for the parameters involved in the various processes plays a crucial role in the color, thickness, and quality of the paper produced. This is an optimization problem where the parameters of the paper manufacturing process are equivalent to the decision variables $X = \{x_1, x_2, \dots, x_d\}$ in the search space and the function $f(X)$ is the fitness or quality function. The value of the fitness function has to be proportional to the paper quality produced. The global optimum of the objective function will be attained when the function has the highest value and the paper produced is of the best quality.

Pattern Recognition

Consider the pattern recognition problem where the objective is to identify and classify objects in an image with 100% accuracy. Every object such as jasmine, rose, car, lorry, bike, building, human face, and fingerprint present in an image has its own set of features that enables recognition and classification. Flowers have features like the number of petals, length and width of the petals, color, diameter of the flower, etc. Similarly a fingerprint has features like ridges and minutiae. The ridge and minutiae patterns on the fingerprint enables classification and recognition.

Feature Reduction

Each object or pattern has its own unique set of features that enables identification. When the number of features of an object is higher it becomes impractical to extract all the features, compare it with the existing template, and then identify the object. So to circumvent the problem, a technique known as feature reduction is applied. This is the technique of selecting a minimum number of features from the entire set, for classification of objects with a reasonable percentage of accuracy. Minimizing the number of features is nothing but the selection of a subset of features, thus reducing the computational complexity in searching for the optimum solution. It is equivalent to minimizing the value of dimension d in the hyperspace. Once d is fixed, the next step is to find the value of the feature set $X = \{x_1, x_2, x_3, \dots, x_d\}$ so that $f(X)$ attains the global maximum or minimum value, as the case may be. If $f(X)$ represents classification accuracy then it will be maximized whereas if $f(X)$ is a classification error then it has to be minimized.

1.5 Formulation of Optimization Problem

The first step in any problem solving is definition of the problem statement. Once the statement is clear, the objectives and constraints have to be outlined. The objective(s) of the problem have to be mathematically formulated as an objective function that is to be either maximized or minimized. The next step is to identify the limitations or boundaries of the design variables as equality or inequality constraints and write mathematical equations for them. When the equations are linear and differentiable, with few parameters, the standard classical methods can be applied. When the equations are non-linear and complex or if the objective function and/or constraints have discontinuities and are not differentiable, the classical methods could become intractable, necessitating the use of evolutionary or metaheuristic algorithms. The nature-inspired algorithms that are basically metaheuristic, population-based search algorithms will be a better choice for such complex problems. Many of the heuristic methods use a greedy criterion in accepting a new solution component that might evolve from the previous solution(s), if it is either going to increase the fitness value or decrease the cost. There is also a possibility of converging at a local optimum for such greedy algorithms, which could be overcome by the metaheuristic algorithms that search in parallel and have good diversity properties.

Although several mathematical techniques exist for solving optimization problems, the computational complexity involved might be too high in some of the cases. Certain approximations might reduce the search and computational complexity and help in arriving at the near-optimum solution in finite time for practical applications. When the number of design variables upon which the objective function is dependent is large, the dimensionality of the problem increases. This can be reduced by dimensionality-reduction methods based on the specific optimization problem. Sensitivity analysis of the objective function with respect to the parameters, i.e. change in the objective function value with changes in the design parameters could be done to improve the parameter settings and hence the performance of the algorithm. As another alternative, multilevel optimization could be applied for large complex problems. When the number of design variables and/or constraints becomes large, the optimization problem becomes unmanageable or impractical. Multilevel optimization involves breaking down an optimization problem of large size or dimensions into smaller sub-problems that can be optimized easily. The smaller sub-problems are linked to put together a solution for the larger problem. Yet another approach is parallel processing, where the sub-problems can be run independently in multiple parallel computers to speed up the optimization process.

The problem is to be defined clearly and the physical principles governing the system to be optimized are to be understood thoroughly. The limitations under which the solution is to be found are formulated as constraints. For example, the number of persons cannot be a fractional number, the resistance value cannot be negative, and so on. These restrictions on certain parameters can be formulated as equality or inequality constraints. Based on the mathematical equations, the problem can be categorized as linear, non-linear, integer, quadratic, etc. When probabilistic variables are involved, it is stochastic programming. Necessary and sufficient conditions for optimality have to be identified and the optimum solution must satisfy these conditions. The best solution among all those available is to be selected based on some criterion. The optimum solution gives a maximum performance measure as compared to other non-optimal solutions. Optimization methods can also be classified as direct and indirect. Direct methods make use of objective functions and constraint equations whereas indirect methods use properties of functions. Most of these

approaches are suitable for programming on a computer, and hence scaling or transformation of variables can be accomplished easily.

1.6 Classification of Optimization Algorithms

The optimization algorithms are classified into the traditional or classical methods, evolutionary algorithms, and swarm intelligence algorithms.

Classical Methods: The classical optimization algorithms are applicable to the traditional continuous optimization functions that are differentiable. The traditional methods involve computation of the first- and second-order derivatives of the objective function in order to find maxima or minima. The solution obtained could be local or global optima but the ultimate goal of optimization is to find the global optimum. They are mostly gradient-based methods and provide deterministic solutions to optimization functions within a continuous search space. The simplex method, linear and non-linear programming, Newton's steepest descent method, Lagrangian method, integer and dynamic programming, and the Kuhn–Tucker conditions are famous classical methods, to name a few. Steepest descent is one of the classical optimization techniques for finding the minimum of a unimodal differentiable function. Figure 1.6 illustrates the trajectory of the steepest descent method. In the classical descent methods, the step size plays a major role in approaching the minimum of a unimodal function. The brute force method is a single point direct search method that searches for the optimum point starting from a single point in the search region that is bounded. The search proceeds in steps, and the step size plays a major role in convergence of the algorithm. When the number of parameters is large, the brute force method suffers from the *curse of dimensionality*.

The random walk method overcomes or circumvents the *curse of dimensionality* by searching from randomly generated points in the search space of the objective function.

FIGURE 1.6
Illustration of gradient descent.

The Hooke–Jeeves method is a direction or pattern search since it starts the search from a base point and has separate step size for each coordinate direction. The trial search points are compared, and if a move is found to be better, then further moves are made in the same direction. Thus the step sizes are adaptive in this method which makes it perform better than the earlier two methods. When the functions are multimodal, the starting point of the search plays a vital role in finding the global optimum since there is more than one optimum (local as well as global) point. There are other methods that are called multi-start since they search the solution space starting from multiple different initial points for each search. The search could be direct or derivative-based. Clustering could be applied where clusters of sample points are formed and one point within the cluster serves as the initial point for the search. This is limited to problems with smaller numbers of parameters since it is computationally intensive. An overview of the classical algorithms has been discussed in the following chapter.

Evolutionary Algorithms: Evolutionary algorithms are a class of optimization algorithms that are based on the biological processes of evolution. As the name implies, evolutionary computational methods are designed on the principles of evolution. They are based on the Darwinian theory of *survival of the fittest* and use selection, mating, reproduction, crossover, and mutation operators similar to the biological processes of evolution. Genetic algorithm (GA), genetic programming (GP), and differential evolution (DE) are three of the classical popular evolutionary algorithms that have been discussed in this book. *Darwin's finches* belong to a group of passerine birds that have around 15 different species within their group. They are diversified in the shape, size, and function of their beaks. Long-term study has shown evolutionary changes in their beaks. They are popularly known as *Darwin's finches* because Charles Darwin found and collected them from the Galapagos Islands and they formed the basis for Darwin's theory of evolution and natural selection. Figure 1.7 shows the *Darwin's finches* that played an important role in Darwin conceiving his theory of evolution and are a typical example of evolution.

Additionally, other evolutionary algorithms available are gene expression programming (GEP), evolutionary programming (EP), and evolutionary strategies (ES). Charles Darwin developed the theory of evolution on which these algorithms are all built upon, and they are all population-based. The fundamental goal of the theory or algorithm is survival of the species.

The transfer of a genetic program (genotype) or code from an individual to its progeny is reproduction. When there are mutations, there are differences in the transferred genetic code that are either advantageous or disadvantageous. When there are more individuals than resources, there is competition and the fitter ones tend to survive and reproduce whereas the weaker ones perish. The genotype carries genetic information from the parent to offspring with all the experience undergone by the parent so far. Phenotype is the set of characteristics or properties of the population, which is a manifestation (behavioral expression) of the genotype in a specific environment. In natural evolution, there is non-linear mapping between genotype and phenotype and it is complex. The algorithms that are developed under the evolutionary framework are broadly referred to as evolutionary algorithms. The evolution of the population is determined by the parents, offspring, and the operators, and the algorithms differ from each other based on this combination. Let P be the population size of parents, C be the number of offspring or children, and after recombination the population size is $P + C$. This has to be resized or scaled down to the original size, and the operators determine the mode of resizing. The fitness of every member of the population is evaluated by the objective function, and the overall fitness is determined by the individual fitness values.

FIGURE 1.7

Darwin's finches. [Author: John Gould (Public Domain).]

In evolutionary strategies the search is in parallel (multi-start) from multiple points in the search space. The initial population is created randomly, and parents are selected for recombination to create a population of children. The children are mutated to change their properties or characteristics; either all of them undergo mutation or it is done selectively. The next-generation population members are chosen from the total population consisting of parents and children. This resizing of the population is done based on the fitness values of the individual members.

Swarm Intelligence Algorithms: Computational intelligence (CI) is a subset of machine learning and artificial intelligence that collectively refers to algorithms with intelligence built into them. Most of the evolutionary algorithms and swarm intelligence algorithms have computational intelligence built into them. Some of the popular computational intelligence algorithms are neural networks, fuzzy logic, genetic algorithm, differential evolution, particle swarm optimization, firefly algorithm, ant colony optimization, and other swarm-based algorithms. Nature-inspired (NI) algorithms are developed based on the study of natural processes that could be physical, chemical, or biological and the behavior of animals, birds, and insects. They are mostly population-based and utilize their collective swarm intelligence in arriving at the optimum solution. Swarm intelligence (SI) algorithms have been developed based on the study of swarm behavior, and these swarms possess computational intelligence. Swarm behavior refers to the collective behavior of a group of insects, birds, fish, and animals, and swarm intelligence is the intelligence exhibited by the entire group as a whole. The individual members of the swarm follow simple rules that lead to collective and productive outcomes for the benefit of the entire swarm. They are adaptive and flexible, have good perception capabilities, and live in

FIGURE 1.8

Weaver ants making an emergency bridge between two plants. (Author: Rose Thumboor, CC BY-SA 4.0. <https://creativecommons.org/licenses/by-sa/4.0/deed.en>.)

the environment utilizing the available resources efficiently. They interact within their group as well as with the environment and behave in a self-organized manner. Most of the swarms have a hierarchy within their group and apply principles of division of labor. Their collective intelligence helps them in foraging for food, defending against predators, and societal interactions such as communication among the members of their group for the benefit of the entire swarm. Such studies have motivated the research towards development of swarm intelligence algorithms that are able to solve complex problems efficiently in finite time. Figure 1.8 shows a group of weaver ants making an emergency bridge between two plants that is a manifestation of group dynamism and collective intelligence.

Many of the classical optimization algorithms require the objective function to be a continuous function whose derivative exists. If the function is discrete in nature or if there are numerous parameters in the problem, these algorithms fail. They are not suitable for dealing with functions whose derivatives do not exist or for discrete optimization problems such as in image processing. In such cases, computational intelligence algorithms are best suited to provide the solution. They are mostly designed for searching a solution space trying to find the best solution or a close approximation. This involves some randomness in the algorithm which in itself increases the efficiency of such methods. Searching a solution space is best done using a swarm or a group of particles, and these serve as the basis for nature-inspired algorithms. They are all population-based and reduce the time of searching to find the optimum solution. SI-based techniques are mostly iterative, and they remember the past history in order to find a better solution than the previous best. Remembering the previous solutions involves memory, and many of the SI algorithms do have memory, either short-term or long-term. The algorithm stops either when a stopping criterion is attained or when the predefined maximum number of iterations is reached. The two most important components in swarm intelligence algorithms are intensification and diversification. In intensification, the search is intensified in a particular area near the

previous best solution, whereas in diversification, the search is conducted over a wider area that was previously unexplored, looking for better solutions.

1.7 Traveling Salesman Problem and Knapsack Problem

Optimizing a cost function that is defined on a set of independent variables is a combinatorial optimization problem since it involves finding the right set of independent variables that maximizes or minimizes the function. The problems which generally fall into this category can be divided into two classes – those which are easy to solve and take up less time (can be solved in polynomial time) and those which take up a large amount of time and are practically infeasible to solve, called NP-hard problems. The most famous problem in computer science under the category of NP-hard is the traveling salesman problem (TSP), and other similar problems in this category are the graph partitioning (coloring) problem, job scheduling problem, and network routing problem, to name a few. These combinatorial optimization problems could be solved by the classical optimization algorithms in an infinitely long time or by the heuristic/metaheuristic optimization algorithms in finite short time that produces an optimum solution that closely approximates the actual solution.

The TSP and the knapsack problem are two typical famous examples in computer science for discrete combinatorial optimization. In the TSP, the objective function is the total distance of the Hamiltonian tour and depends on the set of cities visited and the total distance covered which is the sum of the distance of the edges connecting the cities, and in the knapsack problem the combination of the weights of the items loaded into the knapsack and their total cost determines the objective function value. Consider the traveling salesman problem which is a typical combinatorial optimization problem in computer science that has been found to be intractable or NP-hard. There are a set of cities that are interconnected. The cities are represented by nodes in a graph and the connections between the nodes are edges. Each edge has a value associated with it that could represent distance or cost. The problem is for a traveling salesman to visit each of these cities once and only once (without retracing) and go back to the starting city traveling a total minimum distance and hence incurring the minimum cost. The path in the graph traced by the traveling salesman is called the Hamiltonian tour. This is illustrated in Figure 1.9 which shows one possible Hamiltonian tour in the graph.

Consider the *knapsack problem* in computer science which is another typical combinatorial optimization problem. There is a knapsack or rucksack and a set of items each with a value and a weight. The items are represented by x_k with a weight of w_k and value v_k , where $k = 1, 2, \dots, N$, assuming there are N number of different items. The objective is to fill the knapsack with as many items as possible so that the total value of the items is maximum. The items can be included as a whole or in fractions, such as $c_1 x_1$ where $c_1 = 1$ is one instance of the item x_1 , $c_1 = 2$ is two instances of the item x_1 , whereas $c_1 = 0.5$ is one-half of the item x_1 . The constraint is that the maximum weight of all the items put in the knapsack should not exceed its capacity of W_{max} kg. In this problem, the objective function is the total value of all the items in the knapsack which can be formulated as the weighted sum of the values of each item put in the knapsack. The constraint is that the sum of the weights of all the items put in the knapsack should not exceed the limit of W_{max} kg. This is a multi-variable, constrained optimization problem with a single objective function that is

FIGURE 1.9
Hamiltonian tour for TSP.

to be maximized. This is illustrated in Figure 1.10. The objective function is a cost function that is to be maximized, given by:

$$f(X) = c_1v_1 + c_2v_2 + \dots + c_Nv_N \quad (1.5)$$

The constraint is:

$$c_1w_1 + c_2w_2 + \dots + c_Nw_N \leq W_{\max} \quad (1.6)$$

where the coefficients $c_1, c_2, \dots, c_N$ are either integers or fractional numbers.

FIGURE 1.10
Knapsack problem.

1.8 Summary

Any problem to be optimized should be formulated in terms of an objective function and a set of parameters. The formulation can be done either as minimization (cost) or maximization (profit) of the objective function. The algorithms proposed for such optimization problems should be able to handle linear or non-linear, continuous or discrete, differentiable or non-differentiable, unimodal or multimodal, and single- as well as multi-objective functions or search spaces. The algorithms should be able to search for the solutions in parallel (inherent parallelism) and be efficient in handling computationally intensive cost functions. The number of parameters on which the objective function or the solution depends should be as small as possible and easy to control. The algorithm should have a faster rate of convergence, and it should converge to the optimum solution in finite time.

The classical algorithms have been discussed in Chapter 2, and the nature-inspired optimization algorithms in general have been discussed in Chapter 3. Chapters 4 and 5 cover the evolutionary algorithms – genetic algorithm (GA) and genetic programming (GP). The rest of the chapters are devoted to each of the nature-inspired algorithms that are popular, with proven abilities and superiority in finding the optimum solution. The standard benchmark functions used for testing and comparing the performance of these algorithms and typical applications for evaluating these algorithms have been outlined in Chapter 19. The book concludes with a summary in Chapter 20.

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