Continuous Time Dynamical Systems

**State Estimation and Optimal Control
with Orthogonal Functions**

**B. M. Mohan
S. K. Kar**

CRC Press
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Dedication

To my wife, Vijaya Lakshmi,
my son, Divyamsh &
my daughter, Tejaswini

B. M. Mohan

To my parents, Satya Ranjan & Manjushree,
my sister, Sanjeeta,
my wife, Swastika &
my daughters, Santika & the late Ayushi

S. K. Kar

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List of Abbreviations

BPFs	Block pulse functions
CP1s	Chebyshev polynomials of the first kind
CP2s	Chebyshev polynomials of the second kind
GOPs	General orthogonal polynomials
HF's	Haar functions
HPs	Hermite polynomials
ISE	Integral square error
JP's	Jacobi polynomials
LaPs	Laguerre polynomials
LPs	Legendre polynomials
LQE	Linear-quadratic-estimator
LQG	Linear-quadratic-Gaussian
LQR	Linear-quadratic-regulator
MIMO	Multi-input multi-output
NSR	Noise-to-signal ratio
OF's	Orthogonal functions
SCFs	Sine-cosine functions
SCP1s	Shifted Chebyshev polynomials of the first kind
SCP2s	Shifted Chebyshev polynomials of the second kind

SJPs	Shifted Jacobi polynomials
SLPs	Shifted Legendre polynomials
TPBV	Two-point boundary-value
WFs	Walsh functions

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Preface

Orthogonal functions can be broadly classified into three families: the piecewise constant family, the polynomial family, and the sine-cosine family. Each family is classified into different classes of basis functions. Classes such as block-pulse functions, Haar functions and Walsh functions belong to the piecewise constant family while Chebyshev polynomials of the first kind and the second kind, Legendre polynomials, Laguerre polynomials, Hermite polynomials etc. belong to the polynomial family. The techniques of reducing the (differential, integral) calculus of continuous-time dynamical systems to an attractive algebra, approximate in the sense of least squares, emerged in the early 1970s, first with piecewise constant family functions. In subsequent years the use of polynomial family and sine-cosine functions has been demonstrated.

It all began with the Walsh functions application to optimal control problems in 1975, and the block-pulse functions application to state estimation problems in 1984. Since then, block-pulse functions, Legendre polynomials and other classes of basis functions have been applied extensively for state estimation of linear time-invariant systems, and optimal control of varieties of systems such as linear/bilinear/nonlinear, time-invariant/time-varying, and delay-free/delay systems. All these studies show that block-pulse functions and Legendre polynomials have definite computational advantages in comparison to other classes of basis functions, as they can easily lead to recursive algorithms.

The work of the authors and the several interesting contributions of other researchers form the core of this book. It is perhaps the first book devoted exclusively to the application of block-pulse

functions and Legendre polynomials to state estimation and optimal control of continuous-time dynamical systems. It is hoped that this book will be of interest to control students and engineers.

B. M. Mohan

S. K. Kar

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Chapter 1

Introduction

The optimal control problem is introduced first. Then optimal control problems of varieties of systems and their solution via different classes of orthogonal functions (OFs) are discussed in the literature review section. The objectives and contributions of the book are stated. The organization of the book is given in the last section.

1.1 Optimal Control Problem

A particular type of system design problem is the problem of “controlling” a system. The translation of control system design objectives into the mathematical language gives rise to the control problem. The essential elements of the control problem are

- A desired output of the system.
- A set of admissible inputs or “controls.”
- A performance or cost functional which measures the effectiveness of a given “control action.”

The objective of the system is often translated into a requirement on the output. Since “control” signals in physical systems are

usually obtained from equipment which can provide only a limited amount of force or energy, constraints are imposed upon the inputs to the system. These constraints lead to a set of admissible inputs.

Frequently the desired objectives can be attained by many admissible inputs, so the engineer seeks a measure of performance or cost of control which will allow him/her to choose the “best” input. The choice of a mathematical performance functional is a subjective matter. Moreover, the cost functional will depend upon the desired behaviour of the system. Most of the time, the cost functional chosen will depend upon the input and the pertinent system variables. When a cost functional has been decided upon, the engineer formulates his/her control problem as follows:

Determine the admissible inputs which generate the desired output and which optimize the chosen performance measure.

At this point, optimal control theory enters the picture to aid the engineer in finding a solution to his/her control problem. Such a solution, when it exists, is called an optimal control. Optimal control deals with the problem of finding a control law for a given system such that a certain optimality criterion is achieved. An optimal control is a set of differential equations describing the paths of the control variables that minimize the cost functional. The theory of optimal control is concerned with operating a dynamic system at minimum cost. The case where the system dynamics are described by a set of linear differential equations and the cost is described by a quadratic functional is called a linear quadratic problem. One of the main results in the theory is that the solution is provided by the linear-quadratic-regulator (LQR).

In this book different classes of systems are considered with quadratic performance criteria and an attempt is made to find the

optimal control law for each class of systems using OFs that can optimize the given performance criteria.

1.2 Historical Perspective

The Legendre polynomials (LPs) originated from determining the force of attraction exerted by solids of revolution [5], and their orthogonal properties were established by Adrian Marie Legendre during 1784–1790. There is another family of OFs known as piecewise constant basis functions whose functional values are constant within any subinterval of time period. There is a class of complete OFs known as block-pulse functions (BPFs) which is more popular and elegant in the areas of parameter estimation, analysis and control.

Control of linear systems by minimizing a quadratic performance index gives rise to a time-varying gain for the linear state feedback, and this gain is obtained by solving a matrix Riccati differential equation [3]. Probably, Chen and Hsiao, 1975, were the first who applied a class of piecewise constant OFs, i.e. Walsh functions (WFs), obtained a numerical solution of the matrix Riccati equation [6] and found the time-varying gain. Then many researchers started investigating the problems of identification, analysis and control using different classes of OFs. The operational matrix for integration of BPFs was derived [8]. Moreover, it was shown that BPFs are more fundamental than WFs and the structure of the integration operational matrix of BPFs is simpler than that of WFs. In [9] it was shown that optimal control problems could be solved using BPFs with minimal computational effort.

In the last three and a half decades, the OF approach was successfully applied to study varieties of problems in systems and

control [22, 57, 59, 61]. The key feature of OFs is that it converts differential or integral equations into algebraic equations in the sense of least squares. So this approach became quite popular computationally as the dynamical equations of a system can be converted into a set of algebraic equations whose solution leads to the solution of the problem.

Before going into the details of the optimal control problem, we first take a look at the problem of estimation of state variables as the state estimation plays an important role in the context of state feedback control. State feedback control system design requires the knowledge of the state vector of the plant. Sometimes, no state variables or only a few state variables are available for measurement. In such cases an observer, either full order or reduced order depending on the situation, is incorporated to estimate the unknown state variables if the plant is observable. In general, it is estimated using the Luenberger observer [1]. But the Luenberger observer produces erroneous estimates in a noisy environment unless the measurement noise is filtered out. Interestingly, the OF approach has an inherent filtering property [59] as it involves an integration process which has the smoothing effect. As it appears from the literature, two attempts have been made on the state estimation problem by using two different classes of OFs, i.e. BPFs [23] and shifted Chebyshev polynomials of first kind (SCP1s)[39] so far. It is observed that the BPF approach [23] is purely recursive and uses multiple integration. The number of integrations increases as the order of the system increases, i.e for an n^{th} order system the state equation has to be integrated n times, which is computationally not attractive.

Next, coming to the SCP1 approach [39], integration operational matrix of SCP1s is less sparse than that of shifted Legendre polynomials (SLPs). So if we use SLPs to develop algorithms, it will obviously be more elegant computationally. Moreover, state estimation cannot be done via SCP1s in a noisy environment.

The problem of optimal control incorporating observers has been successfully studied via different classes of OFs, namely BPFs [19], SLPs [30, 47], shifted Jacobi polynomials (SJPs) [33], general orthogonal polynomials (GOPs) [41], sine-cosine functions (SCFs) [44, 47], SCP1s [25, 47], shifted Chebyshev polynomials of the second kind (SCP2s) [47] and single-term Walsh series [52]. The approach followed in [25, 30, 33, 41, 44, 47] is nonrecursive, while it is recursive in [19, 52], making the approach in [25, 30, 33, 41, 44, 47] computationally not attractive.

Synthesis of optimal control laws for deterministic systems described by integro-differential equations has been investigated [11] via the dynamic programming approach. Subsequently, this problem has been studied via BPFs [37], SLPs [34, 43], and SCP1s [42].

The linear-quadratic-Gaussian (LQG) control problem [4] concerns linear systems disturbed by additive white Gaussian noise, incomplete state information and quadratic costs. The LQG controller is simply the combination of a linear-quadratic-estimator (LQE), i.e. Kalman filter, and an LQR. The separation principle guarantees that these can be designed and computed independently. In [35] the solution of the LQG control design problem has been obtained by employing GOPs. By using the GOPs the nonlinear Riccati differential equations have been reduced to nonlinear algebraic equations. The set of nonlinear algebraic equations has been solved to get the solutions. The above approach is

neither simple nor elegant computationally, as nonlinear equations are involved.

Singular systems have been of considerable importance as they are often encountered in many areas. Singular systems arise naturally in describing large-scale systems [53]; examples occur in power and interconnected systems. In general, an interconnection of state variable subsystems is conveniently described as a singular system. The singular system is called generalized state-space system, implicit system, semi-state system, or descriptor system. Optimal control of singular systems has been discussed in [15] and [18]. In [38] the necessary conditions for the existence of optimal control have been derived. It has been shown that the optimal control design problem reduces to a two-point boundary-value (TPBV) problem for the determination of the optimal state trajectory. The single-term Walsh series method [56] has been applied to study the optimal control problem of singular systems. In [62] SLPs were used to solve the same problem. However, this approach is nonrecursive in nature. The Haar wavelet approach [68] has been presented to study the optimal control problem of linear singularly perturbed systems. In the recent times, SCFs [71], SCPs [74] and Legendre wavelets [82] have been applied for solving the optimal control problem of singular systems. These approaches are again nonrecursive.

Time-delay systems are those systems in which time delays exist between the application of input or control to the system and its resulting effect on it. They arise either as a result of inherent delays in the components of the system or as a deliberate introduction of time delays into the system for control purposes. Examples of such systems are electronic systems, mechanical systems, biological

systems, environmental systems, metallurgical systems, chemical systems, etc. A few practical examples [46] are controlling the speed of a steam engine running an electric power generator under varying load conditions, and control of room temperature, a cold rolling mill, spaceship, hydraulic system, etc.

As it appears from the literature, extensive work was done on the problem of optimal control of linear continuous-time dynamical systems containing time delays. Palanisamy and Rao [20] appear to be the first to study the optimal control problem via WFs. They considered time-invariant systems with one delay in state and one delay in control. In [24] time-varying systems containing one delay in state and one delay in control were considered, and optimal control problems of such systems was studied via BPFs. Solutions obtained in [20, 24] are piecewise constant. In order to obtain smooth solution, SCP1s [27] were used to study time-invariant systems with a delay in state only. In [31] time-varying systems with multiple delays in state and control have been studied via SLPs. The problem considered in [20] was investigated again by applying SLPs [36]. In [48] the problem investigated in [24] was again solved by approximating the time-delay terms via Pade approximation and using GOPs. Similarly, in [58] the problem considered in [27] was studied again via SLPs. Linear Legendre multiwavelets [81] were used to solve the time-varying systems having a delay in state only.

In recent years, people have come up with a new idea of defining hybrid functions (with BPFs and any class of polynomial functions) and utilizing the same for studying problems in Systems and Control. The so-called hybrid functions approach was first introduced in [67] to study the optimal control problem of time-

varying systems with a time-delay in state only. Subsequently, this approach was extended to time-invariant systems [72] studied in [27], delay systems containing reverse time terms [73], and time-varying systems [83] considered in [67]. In [84] general Legendre wavelets were used to solve the optimal control problem of time-varying singular systems having a delay in state only.

Looking at the historical developments on solving optimal control problem of nonlinear systems via the OF approach, we find that not much work has been reported. Lee and Chang [40] appear to be the first to study the optimal control problem of nonlinear systems using GOPs. For this, they introduced a nonlinear operational matrix of GOPs. Though their work is fundamental and significant, it is not attractive computationally, as it involves the Kronecker product [12] operation. Chebyshev polynomials of first kind (CP1s) were used [49] for solving nonlinear optimal control problems. In [54] a general framework for nonlinear optimal control problems was developed by employing BPFs.

A BPF approach to hierarchical control of linear time-varying large-scale systems has been reported [45]. The resulting solutions are found to be piecewise constant with minimal mean-square-error.

1.3 Organisation of the Book

The book consists of eleven chapters in all. They are

Chapter 1: Introduction.

Chapter 2: Orthogonal Functions and Their Properties; in this chapter brief discussion on OFs and their classification are given. BPFs and their properties are given in Section 2.2.

LPs are presented in Section 2.3. SLPs and their properties are discussed in Section 2.4. A nonlinear operational matrix using LPs and BPFs is introduced in Section 2.5.

Chapter 3: State Estimation; estimation of unknown state variables is done using full order observer, both in noisy and noise free environments. The algorithm in [23] is modified to deal with nonzero initial state vector.

Chapter 4: Linear Optimal Control Systems Incorporating Observers; using a reduced order observer the unknown state variables are estimated and the optimal control law is obtained.

Chapter 5: Optimal Control of Systems Described by Integro-Differential Equations; a unique approach via BPFs or SLPs is presented in Section 5.2 to solve the optimal control problem of linear systems described by integro-differential equations.

Chapter 6: Linear-Quadratic-Gaussian Control; the LQG control design problem is discussed in Section 6.2. Solutions are given in Sections 6.3 and 6.4 which contain a unified approach and two recursive algorithms.

Chapter 7: Optimal Control of Singular Systems; in Section 7.2 a singular system problem is solved as an initial value problem under some conditions and two recursive algorithms are given. A more generalized solution to the problem is given in Section 7.3.

Chapter 8: Optimal Control of Time-Delay Systems; an approach to compute optimal control law of linear time-varying multi-delay dynamic systems with a quadratic performance index is discussed in Section 8.2. Then application of the approach to

- Time-invariant systems
- Delay free systems
- Singular systems with delays

is given in Subsections 8.2.3, 8.2.4 and 8.2.5. In Section 8.3 optimal control of delay systems with reverse time terms is discussed and algorithms are presented.

Chapter 9: Optimal Control of Nonlinear Systems; a new method for computing optimal control law for nonlinear systems is given by employing LPs and BPFs.

Chapter 10: Hierarchical Control of Linear Systems; a BPF method of hierarchical control of linear time-invariant/time-varying large scale systems is presented.

Chapter 11: Epilogue; the last chapter concludes the book with the scope for further research.

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