Linear Operators and Their Essential Pseudospectra

Aref Jeribi

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Apple Academic Press Inc.
3333 Mistwell Crescent
Oakville, ON L6L 0A2 Canada

Apple Academic Press Inc.
9 Spinnaker Way
Waretown, NJ 08758 USA

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Exclusive worldwide distribution by CRC Press, a member of Taylor & Francis Group

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Printed in the United States of America on acid-free paper

International Standard Book Number-13: 978-1-77188-699-4 (Hardcover)

International Standard Book Number-13: 978-1-351-04627-5 (eBook)

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Library and Archives Canada Cataloguing in Publication

Jeribi, Aref, author

Linear operators and their essential pseudospectra / Aref Jeribi, PhD (Department of Mathematics, University of Sfax, Tunisia).

Includes bibliographical references and index.

Issued in print and electronic formats.

ISBN 978-1-77188-699-4 (hardcover).--ISBN 978-1-351-04627-5 (PDF)

1. Linear operators. 2. Spectral theory (Mathematics). I. Title.

QA329.2.J47 2018

515'.7246 C2018-900406-1 C2018-900407-X

.....
CIP data on file with US Library of Congress
.....

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*To
my mother Sania, my father Ali,
my wife Fadoua, my children Adam and Rahma,
my brothers Sofien and Mohamed Amin,
my sister Elhem, and
all members of my extended family*

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Contents

About the Author	xiii
Preface	xv
1 Introduction	1
1.1 Essential Spectra and Relative Essential Spectra	2
1.2 Essential Pseudospectra	8
1.3 Structured Essential Pseudospectra and Relative Structured Essential Pseudospectra	12
1.4 Condition Pseudospectrum	14
1.5 Outline of Contents	15
2 Fundamentals	19
2.1 Operators	19
2.1.1 Linear Operators	19
2.1.2 Bounded Operators	20
2.1.3 Closed and Closable Operators	20
2.1.4 Adjoint Operator	22
2.1.5 Direct Sum	24
2.1.6 Resolvent Set and Spectrum	25
2.1.7 Compact Operators	26
2.1.8 A -Defined, A -Bounded, and A -Compact Operators	27
2.1.9 Weakly Compact and A -Weakly Compact Operators	29
2.1.10 Dunford-Pettis Property	29
2.1.11 Strictly Singular Operators	30

2.1.12	Strictly Cosingular	31
2.1.13	Perturbation Function	32
2.1.14	Measure of Non-Strict-Singularity	33
2.1.15	Semigroup Theory	33
2.2	Fredholm and Semi-Fredholm Operators	34
2.2.1	Definitions	34
2.2.2	Basics on Bounded Fredholm Operators	37
2.2.3	Basics on Unbounded Fredholm Operators	43
2.3	Perturbation	48
2.3.1	Fredholm and Semi-Fredholm Perturbations	48
2.3.2	Semi-Fredholm Perturbations	51
2.3.3	Riesz Operator	52
2.3.4	Some Perturbation Results	53
2.3.5	A-Fredholm Perturbation	54
2.3.6	A-Compact Perturbations	55
2.3.7	The Convergence Compactly	56
2.4	Ascent and Descent Operators	57
2.4.1	Bounded Operators	57
2.4.2	Unbounded Operators	62
2.5	Semi-Browder and Browder Operators	63
2.5.1	Semi-Browder Operators	63
2.5.2	Fredholm Operator with Finite Ascent and Descent	65
2.6	Measure of Noncompactness	67
2.6.1	Measure of Noncompactness of a Bounded Subset	68
2.6.2	Measure of Noncompactness of an Operator	69
2.6.3	Measure of Non-Strict-Singularity	71
2.6.4	γ -Relatively Bounded	73
2.6.5	Perturbation Result	73
2.7	γ -Diagonally Dominant	75
2.8	Gap Topology	76
2.8.1	Gap Between Two Subsets	76
2.8.2	Gap Between Two Operators	77

2.8.3	Convergence in the Generalized Sense	78
2.9	Quasi-Inverse Operator	79
2.10	Limit Inferior and Superior	83
3	Spectra	85
3.1	Essential Spectra	85
3.1.1	Definitions	85
3.1.2	Characterization of Essential Spectra	88
3.2	The Left and Right Jeribi Essential Spectra	91
3.3	S -Resolvent Set, S -Spectra, and S -Essential Spectra	92
3.3.1	The S -Resolvent Set	92
3.3.2	S -Spectra	98
3.3.3	S -Browder's Resolvent	100
3.3.4	S -Essential Spectra	104
3.4	Invariance of the S -Essential Spectrum	108
3.5	Pseudospectra	113
3.5.1	Pseudospectrum	113
3.5.2	S -Pseudospectrum	115
3.5.3	Ammar-Jeribi Essential Pseudospectrum	119
3.5.4	Essential Pseudospectra	122
3.5.5	Conditional Pseudospectrum	123
3.6	Structured Pseudospectra	124
3.6.1	Structured Pseudospectrum	124
3.6.2	The Structured Essential Pseudospectra	125
3.6.3	The Structured S -Pseudospectra	126
3.6.4	The Structured S -Essential Pseudospectra	128
4	Perturbation of Unbounded Linear Operators by γ-Relative Boundedness	129
4.1	Sum of Closable Operators	129
4.1.1	Norm Operators	129
4.1.2	Kuratowski Measure of Noncompactness	135
4.2	Block Operator Matrices	137

4.2.1	2×2 Block Operator Matrices	137
4.2.2	3×3 Block Operator Matrices	139
5	Essential Spectra	143
5.1	Characterization of Essential Spectra	143
5.1.1	Characterization of Left and Right Weyl Essential Spectra	143
5.1.2	Characterization of Left and Right Jeribi Essential Spectra	149
5.2	Stability of Essential Approximate Point Spectrum and Essential Defect Spectrum of Linear Operator	156
5.2.1	Stability of Essential Spectra	156
5.2.2	Invariance of Essential Spectra	159
5.3	Convergence	162
5.3.1	Convergence Compactly	162
5.3.2	Convergence in the Generalized Sense	169
6	S-Essential Spectra of Closed Linear Operator on a Banach Space	183
6.1	S -Essential Spectra	183
6.1.1	Characterization of S -Essential Spectra	183
6.1.2	Stability of S -Essential Spectra of Closed Linear Operator	188
6.2	S -Left and S -Right Essential Spectra	195
6.2.1	Stability of S -Left and S -Right Fredholm Spectra	195
6.2.2	Stability of S -Left and S -Right Browder Spectra	201
7	S-Essential Spectrum and Measure of Non-Strict-Singularity	205
7.1	A Characterization of the S -Essential Spectrum	205
7.2	The S -Essential Spectra of 2×2 Block Operator Matrices	210
8	S-Pseudospectra and Structured S-Pseudospectra	217
8.1	Study of the S -Pseudospectra	217
8.2	Characterization of the Structured S -Pseudospectra	223

8.3	Characterization of the Structured S -Essential Pseudospectra	231
9	Structured Essential Pseudospectra	239
9.1	On a Characterization of the Structured Wolf, Ammar-Jeribi, and Browder Essential Pseudospectra	239
9.1.1	Structured Ammar-Jeribi, and Browder Essential Pseudospectra	239
9.1.2	A Characterization of the Structured Browder Essential Pseudospectrum	252
9.2	Some Description of the Structured Essential Pseudospectra	262
9.2.1	Relationship Between Structured Jeribi and Structured Ammar-Jeribi Essential Pseudospectra	262
9.2.2	A Characterization of the Structured Ammar-Jeribi Essential Pseudospectrum	264
10	Structured Essential Pseudospectra and Measure of Noncompactness	269
10.1	New Description of the Structured Essential Pseudospectra	269
10.1.1	A Characterization of the Structured Ammar-Jeribi Essential Pseudospectrum by Kuratowski Measure of Noncompactness	269
10.1.2	A Characterization of the Structured Browder Essential Pseudospectrum by Means of Measure of Non-Strict-Singularity	274
11	A Characterization of the Essential Pseudospectra	279
11.1	Approximation of ε -Pseudospectrum	279
11.2	A Characterization of Approximation Pseudospectrum . . .	283
11.3	Essential Approximation Pseudospectrum	293
11.4	Properties of Essential Pseudospectra	298
11.5	Pseudospectrum of Block Operator Matrices	305

12 Conditional Pseudospectra	315
12.1 Some Properties of $\Sigma^\varepsilon(A)$	315
12.2 Characterization of Condition Pseudospectrum	322
Bibliography	327
Index	345

About the Author

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Aref Jeribi, PhD, is Professor in the Department of Mathematics at the University of Sfax, Tunisia. He is the author of the book *Spectral Theory and Applications of Linear Operators and Block Operator Matrices* (2015) and co-author of the book *Nonlinear Functional Analysis in Banach Spaces and Banach Algebras: Fixed Point Theory under Weak Topology for Nonlinear Operators and Block Operator Matrices with Applications* (CRC Press, 2015). He has published many journal articles in international journals. His areas of interest include spectral theory, matrice operators, transport theory, Gribov operator, Bargman space, fixed point theory, Riesz basis, and linear relations.

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Preface

This book is intended to provide a fairly comprehensive study of spectral theory of linear operators defined on Banach spaces. The central items of interest include various essential spectra, but we also consider some of their generalizations that have been studied in recent years.

As the spectral theory of operators is an important part of functional analysis and has numerous applications in many parts of mathematics and, we require, in this book, some modest prerequisites from functional analysis and operator theory are necessary that the reader can find in the classical texts of functional analysis. We therefore have the hope that this book is accessible to newcomers and graduate students of mathematics with a standard background in analysis.

A considerable part of the content of this book corresponds to research activities developed in collaboration with some colleagues as well as with some of my graduate students over the course of several years. This book is considered as an attempt to organize the available material, most of which exists only in the form of research papers scattered throughout the literature. For this reason, it has been a great pleasure for me to organize the material in this book.

In recent years, spectral theory has witnessed an explosive development. In this book, we present a survey of results concerning various types of essential spectra and pseudospectra in a unified, axiomatic way, and we also gathered several topics that are new but that relate only to those concepts and methods emanating from other parts of the book.

The book covers an important list of topics in spectral theory and will be an excellent choice. It is well written, giving a survey of results

concerning various types of spectra in a unified, axiomatic way. The main topics include essential spectra, essential pseudospectra, structured essential pseudospectra, and their relative sets.

We do hope that this book will be very useful for researchers, since it represents not only a collection of a previously heterogeneous material but is also an innovation through several extensions.

Of course, it is impossible for a single book to cover such a huge field of research. In making personal choices for inclusion of material, we tried to give useful complementary references in this research area, hence probably neglecting some relevant works. We would be very grateful to receive any comments from readers and researchers, providing us with some information concerning some missing references.

We would like to thank Salma Charfi for the improvement she has made in the introduction of this book. We are indebted to her. We would also like to thank Aymen Ammar for the improvement he has made throughout this book, and we are very grateful to him. Moreover, we should mention that the thesis work, performed under my direction, by my former students and present colleagues Asrar Elleuch, Mohamemd Zerai Dhahri, Bilel Boukettaya, Kamel Mahfoudhi, and Faten Bouzayeni, obtained results that have helped us in writing this book. Finally, we apologize in case we have forgotten to quote any author who has contributed, directly or indirectly, to this work.

—Aref Jeribi, PhD

Chapter 1

Introduction

Recently, the interest in description of essential spectra remains high because of the abundance of practical applications that help scientists to deal with information overload. This book is intended to provide an important list of topics in spectral theory of linear operators defined on Banach spaces. Central items of interest include Fredholm operators and various characterizations of essential spectra.

In this book, a survey of the state of the art of research related to essential spectra of closed, densely defined, and linear operators subjected to various perturbations is outlined. As important supersets of the essential spectra, the description of essential pseudospectra in this book are interesting objects by themselves, since they carry more information and have a better convergence and approximation properties than essential spectra. We are also interested in giving a much better insight into the essential pseudospectra, by studying some sets called structured essential pseudospectra. Further, a significant amount of research has been done in this book to treat important characterizations of S -essential spectra and S -structured essential pseudospectra.

In this book, we also turn our attention to the important concept of condition pseudospectrum which carries more information than spectrum and pseudospectrum, especially, about transient instead of just asymptotic behavior of dynamical systems.

Now, let us describe its contents in the following sections.

1.1 ESSENTIAL SPECTRA AND RELATIVE ESSENTIAL SPECTRA

The theory of the essential spectra of linear operators in Banach spaces is a modern section of the spectral analysis. It has numerous applications in many parts of mathematics and physics including matrix theory, function theory, complex analysis, differential and integral equations, and control theory.

The original definition of the essential spectrum goes back to H. Weyl [158] around 1909, when he defined the essential spectrum for a self-adjoint operator A on a Hilbert space as the set of all points of the spectrum of A that are not isolated eigenvalues of finite algebraic multiplicity and he proved that the addition of a compact operator to A does not affect the essential spectrum. Irrespective of whether A is bounded or not on a Banach space X , there are many ways to define the essential spectrum, most of them are enlargement of the continuous spectrum. Hence, we can find several definitions of the essential spectrum in the literature, which coincide for self-adjoint operators on Hilbert spaces (see, for example, [79, 137]).

On the other hand, the concept of essential spectra was introduced and studied by many mathematicians and we can cite the contributions of H. Weyl and his collaborators (see, for instance [1–3, 48, 82, 83, 108, 120, 141, 154, 158]). Further important characterizations concerning essential spectra and their applications to transport operators are established by A. Jeribi and his collaborators (see Refs. [1, 4–9, 11–14, 19, 26, 31, 34, 36, 37, 40, 41, 43–47, 51, 53, 59, 60, 84–88, 90, 94, 95, 97–100, 114–116, 123]).

Among these essential spectra, the following sets are defined for a closed densely defined linear operator A :

$$\begin{aligned}
\sigma_{e1}(A) &:= \{\lambda \in \mathbb{C} \text{ such that } \lambda - A \notin \Phi_+(X)\}, \\
\sigma_{e2}(A) &:= \{\lambda \in \mathbb{C} \text{ such that } \lambda - A \notin \Phi_-(X)\}, \\
\sigma_{e3}(A) &:= \sigma_{e1}(A) \cap \sigma_{e2}(A), \\
\sigma_{e4}(A) &:= \{\lambda \in \mathbb{C} \text{ such that } \lambda - A \notin \Phi(X)\}, \\
\sigma_{e5}(A) &:= \bigcap_{K \in \mathcal{K}(X)} \sigma(A + K), \\
\sigma_{e6}(A) &:= \mathbb{C} \setminus \left\{ \lambda \in \mathbb{C} \text{ such that } \lambda - A \text{ is Fredholm, } i(\lambda - A) = 0, \right. \\
&\quad \left. \text{and all scalars near } \lambda \text{ are in } \rho(A) \right\}, \\
\sigma_{e7}(A) &:= \bigcap_{K \in \mathcal{K}(X)} \sigma_{ap}(A + K), \\
\sigma_{e8}(A) &:= \bigcap_{K \in \mathcal{K}(X)} \sigma_{\delta}(A + K), \\
\sigma_{ewl}(A) &:= \bigcap_{K \in \mathcal{K}(X)} \sigma_l(A + K), \\
\sigma_{ewr}(A) &:= \bigcap_{K \in \mathcal{K}(X)} \sigma_r(A + K),
\end{aligned}$$

where $\Phi_+(X)$, $\Phi_-(X)$, and $\Phi(X)$ denote the sets of upper semi-Fredholm, lower semi-Fredholm, and Fredholm operators, respectively; $\rho(A)$, $\sigma(A)$, and $i(A)$ denote the resolvent set, the spectrum, and the index of A , respectively; $\mathcal{K}(X)$ denotes the set of all compact linear operators on X , and

$$\begin{aligned}
\sigma_{ap}(A) &:= \left\{ \lambda \in \mathbb{C} \text{ such that } \inf_{\|x\|=1, x \in \mathcal{D}(A)} \|(\lambda - A)x\| = 0 \right\}, \\
\sigma_{\delta}(A) &:= \left\{ \lambda \in \mathbb{C} \text{ such that } \lambda - A \text{ is not surjective} \right\}, \\
\sigma_l(A) &:= \left\{ \lambda \in \mathbb{C} \text{ such that } \lambda - A \text{ is not left invertible} \right\}, \\
\sigma_r(A) &:= \left\{ \lambda \in \mathbb{C} \text{ such that } \lambda - A \text{ is not right invertible} \right\}.
\end{aligned}$$

The sets $\sigma_{e1}(\cdot)$ and $\sigma_{e2}(\cdot)$ are the Gustafson and Weidmann essential spectra, respectively [79]; $\sigma_{e3}(\cdot)$ is the Kato essential spectrum [108]; the subset $\sigma_{e4}(\cdot)$ is the Wolf essential spectrum [160]; $\sigma_{e5}(\cdot)$ is the Schechter essential spectrum [86–88]; and $\sigma_{e6}(\cdot)$ denotes the Browder essential spectrum [48, 96]. The subset $\sigma_{e7}(\cdot)$ was introduced by V. Racočević [133] and designates the essential approximate point spectrum, and $\sigma_{e8}(\cdot)$ is

the essential defect spectrum and was introduced by C. Shmoeger [143]. $\sigma_{ewl}(\cdot)$ and $\sigma_{ewr}(\cdot)$ are the left and the right of Weyl essential spectra, respectively.

Considerable attention has also been devoted by A. Jeribi [1,93,113] to give a new characterization of the Schechter essential spectrum named the Jeribi essential spectrum, $\sigma_j(A)$, of $A \in \mathcal{C}(X)$ (the set of closed densely defined linear operators on X) defined by

$$\sigma_j(A) = \bigcap_{K \in \mathcal{W}^*(X)} \sigma(A + K)$$

where $\mathcal{W}^*(X)$ stands for each one of the sets $\mathcal{W}(X)$ (the set of weakly compact operators) or $\mathcal{S}(X)$ (the set of strictly singular operators).

Recently, an important progress has been made by A. Ammar, B. Boukettaya, and A. Jeribi [20], who were interested in studying the stability problem of the left and right Weyl operator sets; and they defined the left and right Jeribi essential spectra for a closed densely defined operator A as:

$$\sigma_j^l(A) = \bigcap_{K \in \mathcal{W}^*(X)} \sigma_l(A + K)$$

and

$$\sigma_j^r(A) = \bigcap_{K \in \mathcal{W}^*(X)} \sigma_r(A + K).$$

At the first sight, $\sigma_{ewl}(A)$ (resp. $\sigma_{ewr}(A)$) and $\sigma_j^l(A)$ (resp. $\sigma_j^r(A)$) seem to be not equal. However, the authors has proved that $\sigma_{ewl}(A) = \sigma_j^l(A)$ and $\sigma_{ewr}(A) = \sigma_j^r(A)$ in L_1 -spaces or X satisfies the Dunford-Pettis property; and hence $\sigma_{ewl}(A)$ and $\sigma_{ewr}(A)$ may be viewed as an extension of $\sigma_j^l(A)$ and $\sigma_j^r(A)$, respectively.

On the other hand, the question of stability of the left and the right essential Weyl spectra was well-treated in Ref. [20] by using the concept of polynomially Riesz operators in order to give a refinement on the definition of these essential spectra via this concept and to show that compactness condition can be relaxed in a very general Banach space setting.

Motivated by the notion of measure of noncompactness, A. Ammar, M. Z. Dhahri and A. Jeribi [23] are interested in the description of the essential approximate point spectrum and the essential defect spectrum of a closed densely defined linear operator by means of upper semi-Fredholm and lower semi-Fredholm operators, respectively.

In literature, the spectral theory of linear operators have been enriched by S. Goldberg [75], who presented a development of some powerful methods for the study of the convergence of a sequence of linear operators in a Banach space and the investigation of the convergence compactly to zero. Recently, A. Ammar and A. Jeribi are concerned in work [32], not only with the case of bounded linear operators $(A_n)_n$ converging compactly to a bounded operator A , but also with the case of a sequence of closed linear operators $(A_n)_n$ converging in the generalized sense to a closed linear operator A .

The notion of generalized convergence was approached as a generalization of convergence in norm for possibly unbounded linear operators, as well as a reliable method for comparing operators. Further, this notion make the authors generalize the results of T. Kato [108], which essentially represents the convergence between their graphs in a certain distance. The obtained results were exploited in Ref. [27] to examine the relationship between the various essential spectrum of A_n and A , where $(A_n)_n$ converges in the generalized sense to A .

In the last years, there have been many studies of the operators pencils, $\lambda S - A$, $\lambda \in \mathbb{C}$ (operator-valued functions of a complex argument) (see, for example, [119, 147]). It is known that many problems of mathematical physics (e.g., quantum theory, transport theory, ...) are reduced to the study of the essential spectra of $\lambda S - A$. For this, it seems interesting to study the following S -essential spectra:

$$\begin{aligned}
\sigma_{e1,S}(A) &:= \{\lambda \in \mathbb{C} \text{ such that } \lambda S - A \notin \Phi_+(X)\}, \\
\sigma_{e2,S}(A) &:= \{\lambda \in \mathbb{C} \text{ such that } \lambda S - A \notin \Phi_-(X)\}, \\
\sigma_{e3,S}(A) &:= \{\lambda \in \mathbb{C} \text{ such that } \lambda S - A \notin \Phi_+(X) \cup \Phi_-(X)\}, \\
\sigma_{e4,S}(A) &:= \{\lambda \in \mathbb{C} \text{ such that } \lambda S - A \notin \Phi(X)\}, \\
\sigma_{e5,S}(A) &:= \{\lambda \in \mathbb{C} \text{ such that } \lambda S - A \notin \mathcal{W}(X)\}, \\
\sigma_{b,S}(A) &:= \{\lambda \in \mathbb{C} \text{ such that } \lambda S - A \notin \mathcal{B}(X)\}, \\
\sigma_{el,S}(A) &:= \{\lambda \in \mathbb{C} \text{ such that } \lambda S - A \notin \Phi_l(X)\}, \\
\sigma_{er,S}(A) &:= \{\lambda \in \mathbb{C} \text{ such that } \lambda S - A \notin \Phi_r(X)\}, \\
\sigma_{ewl,S}(A) &:= \{\lambda \in \mathbb{C} \text{ such that } \lambda S - A \notin \mathcal{W}_l(X)\}, \\
\sigma_{ewr,S}(A) &:= \{\lambda \in \mathbb{C} \text{ such that } \lambda S - A \notin \mathcal{W}_r(X)\}, \\
\sigma_{bl,S}(A) &:= \{\lambda \in \mathbb{C} \text{ such that } \lambda S - A \notin \mathcal{B}^l(X)\}, \\
\sigma_{br,S}(A) &:= \{\lambda \in \mathbb{C} \text{ such that } \lambda S - A \notin \mathcal{B}^r(X)\}, \\
\sigma_{eap,S}(A) &:= \left\{ \lambda \in \mathbb{C} \text{ such that } \lambda S - A \notin \Phi_+(X) \text{ and } i(\lambda S - A) \leq 0 \right\}, \\
\sigma_{e\delta,S}(A) &:= \left\{ \lambda \in \mathbb{C} \text{ such that } \lambda S - A \notin \Phi_-(X) \text{ and } i(\lambda S - A) \geq 0 \right\},
\end{aligned}$$

where $\mathcal{B}(X)$ is the set of Riesz-Schauder operators; $\Phi_l(X)$ is the set of left Fredholm operators; $\Phi_r(X)$ is the set of right Fredholm operators; $\mathcal{W}_l(X)$ is the set of left Weyl operators; $\mathcal{W}_r(X)$ is the set of right Weyl operators; $\mathcal{B}^l(X)$ is the set of left Browder operators; and $\mathcal{B}^r(X)$ is the set of right Browder operators. They can be ordered as

$$\sigma_{e3,S}(A) = \sigma_{e1,S}(A) \cap \sigma_{e2,S}(A) \subseteq \sigma_{e4,S}(A) \subseteq \sigma_{e5,S}(A) \subseteq \sigma_{b,S}(A),$$

$$\sigma_{el,S}(A) \subseteq \sigma_{ewl,S}(A) \subseteq \sigma_{bl,S}(A),$$

$$\sigma_{er,S}(A) \subseteq \sigma_{ewr,S}(A) \subseteq \sigma_{br,S}(A).$$

In Ref. [29], the authors proved that

$$\sigma_{e4,S}(A) = \sigma_{el,S}(A) \cup \sigma_{er,S}(A), \quad \sigma_{e1,S}(A) \subset \sigma_{el,S}(A) \text{ and } \sigma_{e2,S}(A) \subset \sigma_{er,S}(A).$$

Note that if $S = I$, we recover the usual definition of the essential spectra of a bounded linear operator A defined in the first section of this introduction. These relative essential spectra drew the attention of Jeribi and his collaborators in Refs. [1, 10, 20, 29, 101].

More precisely, the authors A. Ammar, B. Boukettaya, and A. Jeribi pursued the analysis started in Refs. [1, 163, 164] for S -left-right essential spectra and they studied the invariance of $\sigma_{ei,S}(\cdot)$ ($i = l, r, wl, wr$) by some class of perturbations and extended a part of the results obtained in Ref. [98] to a large class of perturbation operators $\mathcal{R}(X)$, which contains $\mathcal{F}(X)$ for the S -left and S -right spectrum, where $\mathcal{R}(X)$ (resp. $\mathcal{F}(X)$) denotes the set of Riesz operators (resp. Fredholm perturbations). They applied their obtained results to describe the S -essential spectra of an integro-differential operator with abstract boundary conditions acting in the Banach space.

At the same time, an important progress has been made in order to describe various relative essential spectra and we can cite in this context the contribution of the work of A. Ammar, M. Z. Dhahri and A. Jeribi in Ref. [25], where the authors are inspired by the work [10] and studied some types of S -essential spectra of linear bounded operators on a Banach space X . More precisely, in Ref. [25], we find a detailed treatment of some subsets of S -essential spectra of closed linear operators by means of the measure of non-strict-singularity.

When dealing with block operator matrices, we recall that the papers [39, 52, 54–56, 103, 111, 120, 146] are concerned with the study of the $\mathcal{I}$ -essential spectra of operators defined by a 2×2 block operator matrix that

$$\mathcal{L}_0 = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

acts on the product $X \times Y$ of Banach spaces, where $\mathcal{I}$ is the identity operator defined on the product space $X \times Y$ by

$$\mathcal{I} = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}.$$

Inspired by the works [39, 111, 120, 146], the aim of the authors in Ref. [101] was to generalize the previous work and they considered a bounded operator S formally defined on the product Banach space $X \times Y$ as

$$S = \begin{pmatrix} M_1 & M_2 \\ M_3 & M_4 \end{pmatrix}.$$

A considerable attention has been also devoted in Ref. [24] in order to give the characterization of the S -essential spectra of the 2×2 matrix operator $\mathcal{L}_0$ acting on a Banach space are given by using the notion of measure of non-strict-singularity. These results are considered as generalizations of the paper of N. Moalla in Ref. [122], where S -essential spectra of some 2×2 operator matrices on $X \times X$ are discussed with $S = I$.

1.2 ESSENTIAL PSEUDOSPECTRA

In 1967, J. M. Varah [155] introduced the first idea of pseudospectra. In 1986, J. H. Wilkinson [159] came up with the modern interpretation of pseudospectrum where he defined it for an arbitrary matrix norm induced by a vector norm. Throughout the 1990s, L. N. Trefethen [135, 150–152] not only initiated the study of pseudospectrum for matrices and operators, but also he talked of approximate eigenvalues and pseudospectrum and used this notion to study interesting problems in mathematical physics. By the same token, several authors worked on this field. For example, we may refer to E. B. Davies [58], A. Harrabi [80], A. Jeribi [1], E. Sharгородsky [145], and M. P. H. Wolff [161] who had introduced the term approximation pseudospectrum for linear operators.

Pseudospectra are interesting objects by themselves since they carry more information than spectra, especially about transient instead of just asymptotic behavior of dynamical systems. Also, they have better convergence and approximation properties than spectra. The definition of pseudospectra of a closed densely defined linear operator A , for every $\varepsilon > 0$, is given by:

$$\sigma_\varepsilon(A) := \sigma(A) \cup \left\{ \lambda \in \mathbb{C} \text{ such that } \|(\lambda - A)^{-1}\| > \frac{1}{\varepsilon} \right\},$$

or by

$$\Sigma_{\varepsilon}(A) := \sigma(A) \cup \left\{ \lambda \in \mathbb{C} \text{ such that } \|(\lambda - A)^{-1}\| \geq \frac{1}{\varepsilon} \right\}.$$

By convention, we write $\|(\lambda - A)^{-1}\| = \infty$ if $\lambda \in \sigma(A)$, (spectrum of A). For $\varepsilon > 0$, it can be shown that $\sigma_{\varepsilon}(A)$ is a larger set and is never empty. The pseudospectra of A are a family of strictly nested closed sets, which grow to fill the whole complex plane as $\varepsilon \rightarrow \infty$ (see [80, 151, 152]). From these definitions, it follows that the pseudospectra associated with various ε are nested sets. Then, for all $0 < \varepsilon_1 < \varepsilon_2$, we have

$$\sigma(A) \subset \sigma_{\varepsilon_1}(A) \subset \sigma_{\varepsilon_2}(A) \text{ and } \sigma(A) \subset \Sigma_{\varepsilon_1}(A) \subset \Sigma_{\varepsilon_2}(A),$$

and that the intersections of all the pseudospectra are the spectrum,

$$\bigcap_{\varepsilon > 0} \sigma_{\varepsilon}(A) = \sigma(A) = \bigcap_{\varepsilon > 0} \Sigma_{\varepsilon}(A).$$

In [58], E. B. Davies has defined another equivalent of pseudospectrum. One is in terms of perturbations of the spectrum, that is, for any closed operator A , we have

$$\sigma_{\varepsilon}(A) := \bigcup_{\|D\| < \varepsilon} \sigma(A + D).$$

Inspired by the notion of pseudospectra, A. Ammar and A. Jeribi in their works [29], thought to extend these results for essential spectra of densely closed linear operators on a Banach space. They declared the new concept of the essential pseudospectra of densely closed linear operators on a Banach space. Because of the existence of several essential spectra, they were interested in focusing their study on the pseudo-Browder essential spectrum. This set was shown to be characterized in the way one would expect by analogy with essential numerical range. As a consequence, the authors in Ref. [29] located the pseudo-Browder essential spectrum between the essential spectra and the essential numerical range.

More precisely, for $A \in \mathcal{C}(X)$ and for every $\varepsilon > 0$, they defined the pseudo-Browder essential spectrum as follows:

$$\sigma_{e6,\varepsilon}(A) = \sigma_{e6}(A) \bigcup \left\{ \lambda \in \mathbb{C} \text{ such that } \|R_b(A, \lambda)\| > \frac{1}{\varepsilon} \right\},$$

where $R_b(A, \lambda)$ is the Browder resolvent of A . The aim of this concept is to study the existence of eigenvalues far from the Browder essential spectrum and also to search the instability of the Browder essential spectrum under every perturbation. Their study of pseudo-Browder essential spectrum enabled them to determine and localize the Browder essential spectrum of a closed, densely defined linear operator on a Banach space.

In Refs. [1, 21, 29, 30], A. Jeribi and his collaborators pursued their studies about essential pseudospectra and defined the following sets:

$$\begin{aligned} \sigma_{e1,\varepsilon}(A) &:= \{ \lambda \in \mathbb{C} \text{ such that } \lambda - A \notin \Phi_+^\varepsilon(X) \}, \\ \sigma_{e2,\varepsilon}(A) &:= \{ \lambda \in \mathbb{C} \text{ such that } \lambda - A \notin \Phi_-^\varepsilon(X) \}, \\ \sigma_{e3,\varepsilon}(A) &:= \{ \lambda \in \mathbb{C} \text{ such that } \lambda - A \notin \Phi_\pm^\varepsilon(X) \}, \\ \sigma_{e4,\varepsilon}(A) &:= \{ \lambda \in \mathbb{C} \text{ such that } \lambda - A \notin \Phi^\varepsilon(X) \}, \\ \sigma_{e5,\varepsilon}(A) &:= \bigcap_{K \in \mathcal{K}(X)} \sigma_\varepsilon(A + K), \\ \sigma_{eap,\varepsilon}(A) &:= \sigma_{e1,\varepsilon}(A) \bigcup \{ \lambda \in \mathbb{C} : i(\lambda - A - D) > 0, \forall \|D\| < \varepsilon \}, \\ \sigma_{e\delta,\varepsilon}(A) &:= \sigma_{e2,\varepsilon}(A) \bigcup \{ \lambda \in \mathbb{C} : i(\lambda - A - D) < 0, \forall \|D\| < \varepsilon \}, \end{aligned}$$

where $\Phi_+^\varepsilon(X)$ (resp. $\Phi_-^\varepsilon(X)$) denotes the set of upper (resp. lower) pseudo semi-Fredholm operator, $\Phi_\pm^\varepsilon(X)$ denotes the set of pseudo semi-Fredholm operator, and $\Phi^\varepsilon(X)$ denotes the set of pseudo Fredholm operators. Note that if ε tends to 0, we recover the usual definition of the essential spectra of a closed linear operator A defined in the first section of this introduction.

In [161], M. P. H. Wolff has given a motivation to study the essential approximation pseudospectrum. In [33], A. Ammar, A. Jeribi and K. Mahfoudhi showed that the notion of essential approximation pseudospectrum can be extended by devoting our studies to the essential approxima-

tion spectrum. For $\varepsilon > 0$ and $A \in \mathcal{C}(X)$, they define

$$\begin{aligned}\sigma_{\text{eap},\varepsilon}(A) &= \bigcap_{K \in \mathcal{K}(X)} \sigma_{\text{ap},\varepsilon}(A + K), \\ \Sigma_{\text{eap},\varepsilon}(A) &= \bigcap_{K \in \mathcal{K}(X)} \Sigma_{\text{ap},\varepsilon}(A + K),\end{aligned}$$

where

$$\sigma_{\text{ap},\varepsilon}(A) := \left\{ \lambda \in \mathbb{C} \text{ such that } \inf_{x \in \mathcal{D}(A), \|x\|=1} \|(\lambda - A)x\| < \varepsilon \right\},$$

and

$$\Sigma_{\text{ap},\varepsilon}(A) := \left\{ \lambda \in \mathbb{C} \text{ such that } \inf_{x \in \mathcal{D}(A), \|x\|=1} \|(\lambda - A)x\| \leq \varepsilon \right\}.$$

In their work, the authors measure the sensitivity of the set $\sigma_{\text{ap}}(A)$ with respect to additive perturbations of A by an operator $D \in \mathcal{L}(X)$ of a norm less than ε . So, they define the approximation pseudospectrum of A by

$$\sigma_{\text{ap},\varepsilon}(A) = \bigcup_{\|D\| < \varepsilon} \sigma_{\text{ap}}(A + D),$$

and they give a characterization of the essential approximation pseudospectrum $\sigma_{\text{eap},\varepsilon}(\cdot)$ which nicely blends these properties of the essential and the approximation pseudospectra, and accordingly the authors are interested in the following essential approximation spectrum

$$\sigma_{\text{eap}}(A) := \bigcap_{K \in \mathcal{K}(X)} \sigma_{\text{ap}}(A + K).$$

As a consequence, the authors obtain the following result

$$\sigma_{\text{eap},\varepsilon}(A) = \bigcup_{\|D\| < \varepsilon} \sigma_{\text{eap}}(A + D).$$

1.3 STRUCTURED ESSENTIAL PSEUDOSPECTRA AND RELATIVE STRUCTURED ESSENTIAL PSEUDOSPECTRA

Structured pseudospectra are very useful tools in control theory and other fields [81]. In particular, D. Hinrichsen and B. Kelb [81] presented in 1993 a graphical method to determine and visualize the spectral value sets of a matrix A .

We also refer the reader to E. B. Davies [58] who defined the structured pseudospectra, or spectral value sets of a closed densely defined linear operator A on a Banach space X by

$$\sigma(A, B, C, \varepsilon) = \bigcup_{\|D\| < \varepsilon} \sigma(A + CDB),$$

where $B \in \mathcal{L}(X, Y)$ (the set of bounded linear operators from X into Y) and $C \in \mathcal{L}(Z, X)$.

The notion of structured pseudospectra drew the attention of A. Elleuch and A. Jeribi who have recently been concerned in [67, 68] with the study of essential spectra of an operator A which is subjected to structured perturbation of the form $A \rightsquigarrow A + CDB$, where B, C are given bounded operators and D is unknown disturbance operator satisfies $\|D\| < \varepsilon$ for a given $\varepsilon > 0$. More specifically, inspired by [1, 86–89, 91, 112], where the authors proved the invariance of the Schechter essential spectrum of a closed densely defined linear operators, A. Elleuch and A. Jeribi, in [67, 68], extended the obtained results for the case of structured Ammar-Jeribi essential pseudospectrum. For this analysis, they needed to characterize the structured pseudospectra of bounded operator and they defined the following structured essential pseudospectra of a closed, densely defined linear operator A with respect to the perturbation structure (B, C) ,

where $B \in \mathcal{L}(X, Y)$, $C \in \mathcal{L}(Z, X)$ and uncertainty level $\varepsilon > 0$,

$$\sigma_{e4}(A, B, C, \varepsilon) := \bigcup_{\|D\| < \varepsilon} \sigma_{e4}(A + CDB),$$

$$\sigma_{e5}(A, B, C, \varepsilon) := \bigcup_{\|D\| < \varepsilon} \sigma_{e5}(A + CDB),$$

$$\sigma_{e6}(A, B, C, \varepsilon) := \bigcup_{\|D\| < \varepsilon} \sigma_{e6}(A + CDB).$$

The subset $\sigma_{e4}(A, B, C, \varepsilon)$ is the structured Wolf essential pseudospectrum, $\sigma_{e5}(A, B, C, \varepsilon)$ is the structured Ammar-Jeribi essential pseudospectrum, and $\sigma_{e6}(A, B, C, \varepsilon)$ denotes the structured Browder essential pseudospectrum.

The main purpose of the authors in Ref. [67] consists in studying the structured essential pseudospectra of the sum of two operators in order to improve and extend the results of Refs. [1, 12, 144] which were done for various essential spectra. In addition, the authors described the behavior of structured Ammar-Jeribi essential pseudospectrum and they presented a fine description of this set by means of compact and Fredholm perturbations.

Further, they used in Ref. [68] some criteria in order to prove the stability of the structured Wolf and the structured Ammar-Jeribi essential pseudospectra. They also used the notion of the measure of noncompactness to give a new characterization of the structured Ammar-Jeribi essential pseudospectrum and they presented some properties of the structured Ammar-Jeribi and Browder essential pseudospectra by means of the measure of non-strict-singularity.

On the other hand, based on the definition of the S -essential spectra and the structured S -pseudospectra, A. Elleuch and A. Jeribi, in [67, 68], introduced and characterized the structured S -pseudospectrum of a closed, densely defined linear operator on a Banach space X . They also established

a relationship between the structured S -essential pseudospectra and the S -essential spectra of a bounded linear operator under some conditions.

1.4 CONDITION PSEUDOSPECTRUM

There are several generalizations of the concept of the spectrum in literature such as Ransford spectrum [134], pseudospectrum [1, 80, 152], and condition spectrum [109]. It is natural to ask whether there are any results similar for operator in infinite dimensional Banach spaces for these sets. The concept of condition pseudospectrum is very important since it carries more information than spectra and pseudospectra, especially, about transient instead of just asymptotic behavior of dynamical systems. Also, condition pseudospectrums have better convergence and approximation properties than spectra and pseudospectra. For a detailed study of the properties of the condition spectrum in finite dimensional space or in Banach algebras, we may refer to S. H. Kulkarni and D. Sukumar in Ref. [109], M. Karow in [106] and G. K. Kumar and S. H. Lui in Ref. [110].

The aim of the authors A. Ammar, A. Jeribi and K. Mahfoudhi in [35] is to show some properties of condition pseudospectra of a linear operator A in Banach spaces and reveal the relation between their condition pseudospectrum and the pseudospectrum (resp. numerical range). One of the central questions consists in characterizing the condition pseudospectra of all bounded linear operators acting in a Banach space. More precisely, the authors consider, in their work, one more such extension in terms of the condition number. In a complex Banach space, the condition pseudospectrum of a bounded linear operator A in infinite dimensional Banach spaces is given for every $0 < \varepsilon < 1$ by:

$$\Sigma^\varepsilon(A) := \sigma(A) \cup \left\{ \lambda \in \mathbb{C} \text{ such that } \|\lambda - A\| \|(\lambda - A)^{-1}\| > \frac{1}{\varepsilon} \right\},$$

with the convention that $\|\lambda - A\| \|(\lambda - A)^{-1}\| = \infty$, if $\lambda - A$ is not invertible.

The authors summarize some properties and useful results of the condition pseudospectrum and they characterize it in order to improve lots recent results of Refs. [106, 109, 110] in infinite dimensional Banach spaces.

1.5 OUTLINE OF CONTENTS

Going on to a brief indication of the contents of the individual chapters, we mention, first of all that our book consists of 12 chapters.

We recall in [Chapter 2](#) some definitions, notations, and basic information on both bounded and unbounded Fredholm operators. We also introduce the concept of semigroup theory, compactly convergence, measure of noncompactness, gap topology, and quasi-inverse operator. Basic concepts and classical results are summarized in the first two sections. In the subsequent sections we study the approximate point spectrum, which is one of the most important examples of a spectrum in Banach algebras. The approximate point spectrum is closely related with the notions of removable and non-removable ideals.

We continue [Chapter 3](#) by giving some fundamentals about characterization of various spectra and essential spectra such as left and right Jeribi essential spectra, S -resolvent set, S -spectra, S -essential spectra, pseudospectra and structured pseudospectra.

The aim of [Chapter 4](#) is to investigate the perturbation of unbounded linear operators via the concept of γ -relative boundedness introduced in [92]. In fact, this chapter is a survey of the paper [22] where the authors try to generalize the some results of [61] by using the new concept of the gap and the γ -relative boundedness in order to establish some sufficient conditions imposed to three unbounded linear operators and to obtain the

closedness of their algebraic sum. The obtained results are useful to study some specific properties of both 2×2 and 3×3 block operator matrices.

[Chapter 5](#) addresses the characterization of essential spectra such as left and right Jeribi and Weyl essential spectra. We also study the stability of essential approximate point spectrum and essential defect spectrum of linear operators and we treat the elegant interaction between essential spectra of closed linear operators and the notion of convergence in the generalized sense.

In [Chapters 6](#) and [7](#), we focus on the study of the stability of S -left and S -right essential spectra. Moreover, we give the characterization of the Schechter S -essential spectrum by means of compact and Fredholm perturbations and also via the concept of measure of non-strict-singularity. Finally, illustrate the obtained results to matrix operator.

We concentrate ourselves exclusively in [Chapter 8](#) on the characterization of S -pseudospectra, structured S -pseudospectra, and structured S -essential pseudospectra. Inspired by the definition of essential spectra, we define and characterize in [Chapter 9](#) the structured Ammar-Jeribi, Browder and Wolf, essential pseudospectra and in particular, we treat the relationship between structured Jeribi and structured Ammar-Jeribi essential pseudospectra.

[Chapter 10](#) focuses on a new description of the structured essential pseudospectra. Not only do we give a characterization of the structured Ammar-Jeribi essential pseudospectrum by Kuratowski measure of noncompactness but also we investigate the structured Browder essential pseudospectrum by means of measure of non-strict-singularity.

The aim of [Chapter 11](#) is to investigate the essential approximation pseudospectrum of closed, densely defined linear operators on a Banach space. We refine the notion of the ε -pseudospectrum and we focus on the characterization of the approximation pseudospectrum and the essential approximation pseudospectrum. We also establish some properties of essential pseudospectra. Further, we study the pseudospectrum of block operator matrix.

Finally, in [Chapter 12](#), we summarize some properties and useful results of the condition pseudospectrum and we characterize it in infinite dimensional Banach spaces.

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