LEARNING AND TEACHING EARLY MATH

The third edition of this significant and groundbreaking book summarizes current research on how young children learn mathematics and illustrates how best to develop foundational knowledge to realize more effective teaching.

Using straightforward, practical language, early math experts Douglas Clements and Julie Sarama show how *learning trajectories* help teachers understand children's level of mathematical understanding and lead to better teaching. By focusing on the inherent delight and curiosity behind young children's mathematical reasoning, learning trajectories ultimately make teaching more joyous: helping teachers understand the varying levels of knowledge exhibited by individual students allows them to better meet the learning needs of all children.

This thoroughly revised and contemporary third edition of *Learning and Teaching Early Math* remains the definitive, research-based resource to help teachers understand the learning trajectories of early mathematics and become confident, credible professionals. The new edition draws on numerous new research studies, offers expanded international examples, and includes updated illustrations throughout.

This new edition is closely linked with *Learning and Teaching with Learning Trajectories*—

[LT]^2—an open-access, web-based tool for early childhood educators to learn about how children think and learn about mathematics. Head to LearningTrajectories.org for ongoing updates, interactive games, and practical tools that support classroom learning.

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LEARNING AND TEACHING EARLY MATH

The Learning Trajectories Approach

Third Edition

Douglas H. Clements and Julie Sarama

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We wish to dedicate this book to those who taught us, including the thousands of children and teachers who shared their lives and thinking with us, and to our closest teachers, our families. We also thank our graduate students, post-doctoral fellows, and colleagues who provided valuable feedback.

CONTENTS

PREFACE

> Who dares to teach must never cease to learn.
>
> (John Cotton Dana, 1856–1929)

> Mathematics is, in its way, the poetry of logical ideas.
>
> (Albert Einstein, 1879–1955)

> Think of the biggest number you can. Now add five. Then, imagine if you had that many Twinkies. Wow, that's five more than the biggest number you could come up with!
>
> (Child, age 6)

Everyone knows that effective teaching involves "meeting the students where they are" and helping them build on what they know. But that's easier said than done. Which aspects of math are important, which less so? How do we diagnose what a child knows? How do we build on that knowledge—in which directions, and in what ways?

We believe that "learning trajectories" help answer these questions and help teachers become more effective professionals. Just as importantly, they open up windows to seeing young children and math in new ways, making teaching more joyous because the mathematical reasoning of children is both impressive and delightful.

Learning trajectories have three parts: (a) a specific mathematical goal, (b) a path along which children develop to reach that goal, and (c) a set of instructional activities fine-tuned for each step along said path that help children reach the following step. So, teachers who understand learning trajectories understand the math, the way children think and learn about math, and how to help children learn it better.

Learning trajectories connect research and practice. They connect children to math. They connect teachers to children. They help teachers understand the level of knowledge and thinking of their classes *and* the individuals in their classes as key in serving the needs of all children. (Equity issues are important to us and to the nation. This entire book is designed to help you teach *all* children, but equity concerns are discussed specifically and at length in Chapters 14, 15, and 16.) *Learning and Teaching Early Math* will help you understand the learning trajectories of early math in order to become a quintessential professional.

Learning and teaching, of course, take place in a context. For the last two decades, we have had the honor and advantage of working with several hundred early childhood teachers who have worked with us on creating new ideas for teaching and have invited us into their classrooms to test these ideas with the children in their charge. We wish to share with you a bit about this collaborative work.

Background

In 1998, we began a 4-year project funded by the National Science Foundation (NSF). The purpose of "Building Blocks—Foundations for Mathematical Thinking, Pre-Kindergarten to Grade 2: Research-Based Materials Development" was to create and evaluate math curricula for young children based on a theoretically sound research and development framework. Based on theory and research on early childhood learning and teaching, we determined that Building Blocks' basic approach would be *finding the mathematics in*, and *developing mathematics from, children's activity*. To achieve this, all aspects of the Building Blocks project have been based on learning trajectories. Teachers have found this combination of the Building Blocks' approach and learning trajectories to be a powerful teaching tool.

More than 20 years later, we are still finding new opportunities for exciting research and development in early math. Funding from the U.S. Department of Education's Institute of Education Sciences (IES), National Science Foundation (NSF), Heising-Simons Foundation, Bill & Melinda Gates Foundation, and Office of Special Education Programs (OSEP) has allowed us to work closely with thousands of teachers and tens of thousands of children. All of these agencies and individuals have contributed ideas to this book and its companion. In addition, these projects have increased our confidence that our approach, based on learning trajectories and rigorous empirical testing at every step, can, in turn, make a contribution to all educators in the field of early math. The model for working with educators in all positions—from teachers to administrators, trainers to researchers—has been developed with IES funding to our TRIAD (Technology-enhanced, Research-based Instruction, Assessment, and professional Development)[1] project.

The "Companion" Books

We believe that our successes are due to the people who have contributed to our projects, as well as to our commitment to grounding everything that we have done in research. Because the work has been so drenched in research, we initially decided to publish two books. The companion to the first edition of the present book—*Early Childhood Mathematics Education Research: Learning Trajectories for Young Children* (Sarama & Clements, 2009)—reviews extensively the research underlying our original learning trajectories, emphasizing the research that describes the paths of learning; that is, children's natural progressions in developing the concepts and skills within a certain domain of math (most research citations for these are in the companion book, although we have added recent ones to this edition). The present book describes and illustrates how these learning trajectories can be implemented in the classroom *and* brings the research and the learning trajectories up to date.

What's New and Different about this Edition

Early childhood math education continues to be of great interest, so there are abundant new research and resources, all of which we share in this edition. International work is particularly highlighted. We appreciate the suggestions our readers have provided, and we tried to put every one into practice.

One of the most important enhancements to this book is our incorporation of our new *Learning and Teaching with Learning Trajectories* tool (www.LearningTrajectories.org). Readers can now see videos of children's thinking at *each level* of each topic or learning trajectory, and of *classroom* and home videos of teachers and caregivers helping children *learn* that topic. Along with hundreds of other resources, this will revolutionize the way one can learn about and use learning trajectories. See more about this tool on pp. 11–14.

Reading this Book

In straightforward, no-nonsense language, we summarize what is known about how children learn and how to build on what they know. In Chapter 1, we introduce the topic of math education for very young children. We discuss why people are particularly interested in engaging young children with math. Next, we describe the idea of learning trajectories in detail. We end with an introduction to the Building Blocks project and explain how learning trajectories are at its core.

Most of the following chapters address one math topic, and we describe how children understand and learn about that topic. These descriptions are brief summaries of the more elaborate reviews of the research that can be found in the aforementioned companion book, *Early Childhood Mathematics Education Research: Learning Trajectories for Young Children* (Sarama & Clements, 2009), as well as updates to those bodies of research. Next, we describe how experiences—from the beginning of life—and classroom-based education affect children's learning of the topic. Chapters 2 to 11 then culminate in a detailed description of learning trajectories for the chapter's topic.

Read more than the topic chapters, even if you just want to teach a topic! In the last three chapters, we discuss issues that are important for putting these ideas into practice. In Chapter 14, we describe how children think about math and how their feelings are involved. Equity concerns complete that chapter. In Chapter 15, we discuss the contexts in which early childhood education occurs and the curricula that are used. In Chapter 16, we review what we know about specific instructional practices. The topics of these three chapters are unique to this book. Because there are no corresponding chapters in the companion book for these three important chapters, we review more research in this book. We have made the implications for practitioners clear.

To teach children with different needs, and to teach effectively, make sure you read Chapters 14, 15, and especially 16. Some readers may wish to read those chapters immediately after having read Chapter 1! Whichever way you choose, please know that the learning trajectories that describe children's learning and effective teaching for each topic are only *part* of the story—the other, critical part is found in those final three chapters.

This is not a typical book of "cute teaching ideas." (OK, many of the teaching ideas and activities, and especially children's reactions to them, are very cute!) We believe, however, that it may be the most practical book that you, as a teacher of early math, could read. The many teachers with whom we have worked claim that, once they understood the learning trajectories and ways to implement them in their classrooms, they—and the children they teach—were changed for the better forever. Moreover, they also changed their beliefs, shedding the unfortunate misconceptions that many teachers hold about early math education, such as:

1. Young children are not ready for mathematics education.
2. Mathematics is for some bright kids with mathematics genes.
3. Simple numbers and shapes are enough.
4. Language and literacy are more important than mathematics.
5. Teachers should provide an enriched physical environment, step back, and let the children play.
6. Mathematics should not be taught as a stand-alone subject matter.
7. Assessment in mathematics is irrelevant when it comes to young children.
8. Children learn mathematics only by interacting with concrete objects.
9. Computers are inappropriate for the teaching and learning of mathematics.

(From Sun Lee & Ginsburg, 2009)

Note

1 Like many acronyms, TRIAD *almost* works … we jokingly ask people to accept the "silent p" in "professional development."

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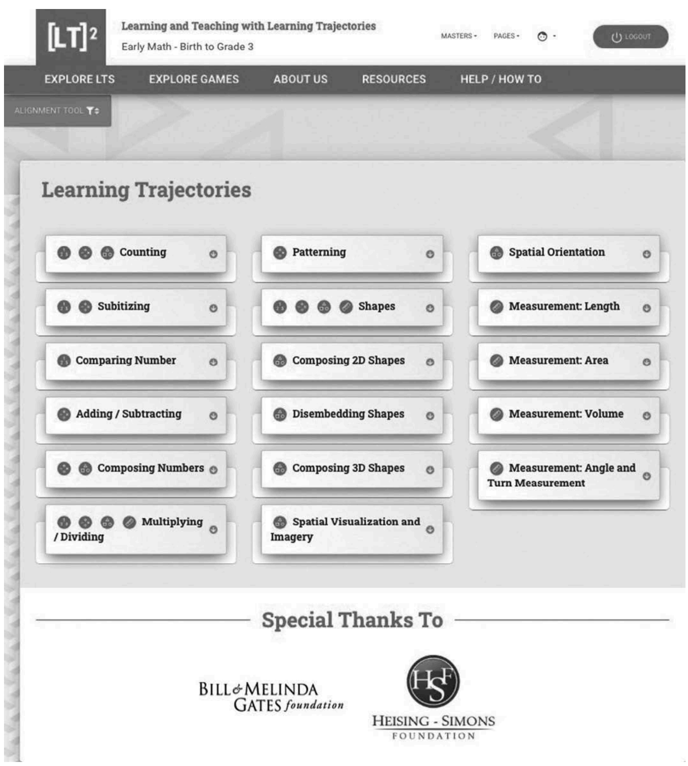
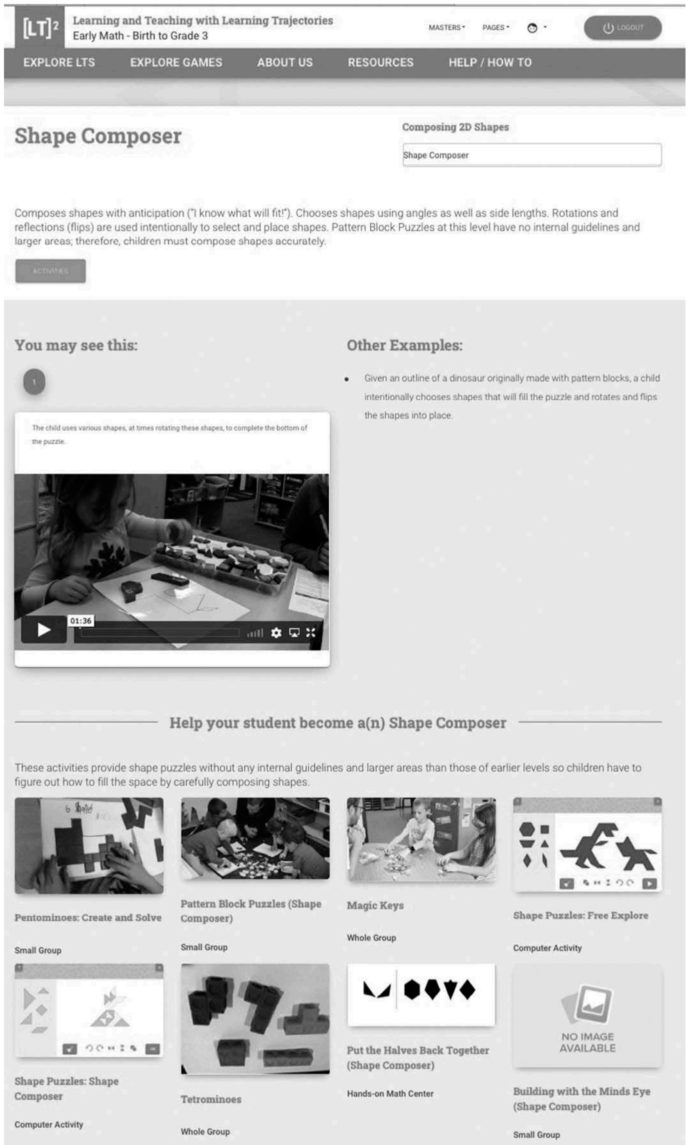
WHAT IS *LEARNING AND TEACHING WITH LEARNING TRAJECTORIES*—[LT]^2?

- [LT]^2 is a web-based tool for early childhood educators to learn about how children think and learn about mathematics, and how to teach mathematics to young children "their way" (birth to age 8).
- [LT]^2 allows teachers, caregivers, and parents to *see* the learning trajectories for math as they view short video clips of classroom instruction and children working on math problems in a way that clearly reveals their thinking.





[LT]^2 is a new open-access tool for early math teaching and learning, *closely linked to this book* and developed thanks to funding from the Heising-Simons Foundation and the Bill and Melinda Gates Foundation, along with decades of research conducted by professors Julie Sarama and Douglas H. Clements. Large-scale studies show that the learning trajectories and [LT]^2 work, as validated by the "What Works Clearinghouse" and praised on the cover of *The New York Times* and in the *Wall Street Journal*. Read about two teachers' use of [LT]^2 – **https://bit.ly/2oQ1Yq4** & **https://bit.ly/2veu83O**.

[LT]^2 runs on all technological platforms, addresses new ages—birth to age 8 years—and includes alignments with standards and assessments, as well as software for children. [LT]^2 enables teachers to help children find the mathematics in—and develop the mathematics from—their everyday activities, including art, stories, puzzles, and games. Head to **Learning Trajectories.org** for on-going updates, interactive games, and practical tools that support classroom learning.

UNIVERSITY of DENVER
MORGRIDGE COLLEGE OF EDUCATION
Marsico Institute for Early Learning & Literacy

[LT]^2

1 Young Children and Mathematics Learning

> Snow was falling in Boston and preschool teacher Sarah Gardner's children were coming in slowly, one bus at a time. She had been doing high-quality math all year, but was still amazed at her children's ability to keep track of the situation: The children kept saying, "Now, 11 are here and 7 absent. Now, 13 are here and 5 absent. Now …."

Why have so many people become interested in math for very young children lately? Because early math is *surprisingly important*.

First, math is increasingly important in a modern global economy, but math achievement in many countries has not kept up. Our own country, the USA, has fewer high-performing and more low-performing students than many other countries, especially in math (http://ncee.org/pisa-2018-lessons/). These differences appear as early as first grade, kindergarten … and even preschool (Gerofsky, 2015b; Mullis, Martin, Foy, & Arora, 2012b; OECD, 2014). Although some high-performing countries are showing improvements, many like the USA are not (Mullis et al., 2012b). This is one reason interest in improving early childhood math education has emerged from around the globe, such as in Africa, South and Latin America, and Asia. These increased interests are often paired with a special focus on children who have not been provided opportunities to learn (McCoy et al., 2018b).

Many young children do not even get the *chance* to learn the more advanced math taught in many other countries. If each child is given such opportunities, *all people in each country benefit*, economically and socially, because everyone contributes more to social and technological advancements.

> *During most of the 20th century, the United States possessed peerless mathematical prowess—not just as measured by the depth and number of the mathematical specialists who practiced here but also by the scale and quality of its engineering, science, and financial leadership, and even by the extent of mathematical education in its broad population. But, without substantial and sustained changes to its educational system, the United States will relinquish its leadership in the 21st century*.
>
> The National Mathematics Advisory Panel[1] (NMP, 2008, p. xi)

Second, these early childhood years have been found to be *surprisingly important for development through life*. That is, what math children know when they *enter* kindergarten predicts their math achievement for years to come (Duncan et al., 2007). Math also predicts later success in *reading* (Duncan et al., 2007; Duncan & Magnuson, 2011), so, *math appears to be a core component of cognition*. Further, knowledge of math in the early years is the best predictor of graduating high school (McCoy et al., 2017; Watts, Duncan, Siegler, & Davis-Kean, 2014). One more argument for early childhood math is that number and arithmetic knowledge at age 7 years predicts socioeconomic status at age 42 (even controlling for all other variables, Ritchie & Bates, 2013).

These predictions may show that *math concepts and skills are important to all of school and life*. Math provides a new way to see the world, the beauty of it, and the way you can solve problems that arise within it. However, math is much more: *Math is critical thinking* and *problem solving*, and high-quality *math experiences also promote social, emotional, literacy, and general brain development* (Aydogan et al., 2005b; Clements, Sarama, Layzer, Unlu, & Fesler, 2020a; Dumas, McNeish, Sarama, & Clements, 2019; Sarama & Clements, 2019b; Sarama, Lange, Clements, & Wolfe, 2012b)! No wonder they predict later success.

Third, although the math-achievements gap between countries is troubling, an even larger and more damaging gap lies between children growing up in higher- and lower-resource communities. Both the income gap and the achievement gap have been increasing for decades (Bachman, Votruba-Drzal, El Nokali, & Castle Heatly, 2015; Reardon, 2011). Children shouldn't be at a disadvantage just because their communities lack resources to provide *charging stations* for learning math—and they do not have to be. They would think and learn just as well if they have the same *opportunities* to learn math early. That's why we are working to make good early math learning resources available to children in all communities.

Fourth, if our country's children have limited math knowledge initially and achieve less later in school compared to children in other countries, can there possibly be bright spots? Yes. From their first years, children have boundless interest and curiosity in math … *and* the ability to learn to think like mathematicians. In high-quality early childhood education programs, young children can engage in surprisingly deep investigations of math ideas. They can learn skills, problem solving, and concepts in ways that are natural and motivating to them. This brings us to the main reason to engage young children in math: *Young children love to think mathematically*. They become exhilarated by their own ideas (like the 6-year-old quoted at the beginning of the preface) and the ideas of others. To develop the *whole* child, we must develop the *mathematical child*.

Fifth, teachers enjoy the reasoning and learning that high-quality math education brings forth from their children. High-quality math throughout early childhood does not involve pushing elementary arithmetic onto younger children. Instead, good education allows children to experience math as they play in and explore their world. A higher proportion of children are in early care and education programs every year. We teachers are responsible for bringing the knowledge and intellectual delight of math to all children, especially those who have not yet had many high-quality educational experiences. Good teachers can meet this challenge with research-based "tools."

Most children acquire considerable knowledge of numbers and other aspects of mathematics before they enter kindergarten. This is important, because the mathematical knowledge that kindergartners bring to school is related to their mathematics learning for years *thereafter—in elementary school, middle school, and even high school. Unfortunately, most children from low-income backgrounds enter school with far less knowledge than peers from middle-income backgrounds, and the achievement gap in mathematical knowledge progressively widens throughout their pre-K-12 years.*

The National Math Advisory Panel (NMP, 2008, p. xvii)

Fortunately, encouraging results have been obtained for a variety of instructional programs developed to improve the mathematical knowledge of preschoolers and kindergartners, especially those from low-income backgrounds. There are effective techniques—derived from scientific research on learning—that could be put to work in the classroom today to improve children's mathematical knowledge.

The National Math Advisory Panel (NMP, 2008, p. xvii)

These tools include specific guidance on how to help children learn in ways that are both appropriate and effective. In this book, we pull that knowledge together to provide a core tool: "learning trajectories" for each major topic in early math.

What are Learning Trajectories?

Children follow natural developmental progressions in learning and development. As a simple example, they learn to crawl, then walk, then run, skip, and jump with increasing speed and dexterity. These are levels in the developmental progression of movement. Children follow natural developmental progressions in learning math, too, by learning math ideas and skills in their own way.

Teachers who understand these developmental progressions for each major domain or topic of math, and base their instruction on them, build math learning environments that are particularly developmentally appropriate, effective, and meaningful (Figure 1.1). These developmental paths are the basis for this book's *learning trajectories*. Learning trajectories help us answer several questions: What goals or objectives should we hold? Where do we start? How do we know where to go next? How do we get there?

Learning trajectories have three parts: (a) a math goal, (b) a developmental path along which children progress to reach that goal, and (c) teaching practices, including the educational environment, interactions, and activities, *matched* to each of the levels of thinking in that path, that help children develop ever-higher levels of thinking. Let's examine each of these three parts.

Goals: The Big Ideas of Math

The first part of a learning trajectory is a math goal. Our goals include the "big ideas of math": clusters of concepts and skills that are mathematically central and coherent, consistent with children's thinking, and generative of future learning. These big ideas come from mathematicians, researchers, and teachers (CCSSO/NGA, 2010; Clements, 2004; NCTM, 2006; NMP, 2008). They include math content but *also research on students' thinking about and learning of math*. As an example, one big idea is that *counting can be used to find out how many in a collection*.

Figure 1.1 Carmen Brown encourages a preschooler to “mathematize”

Development Progressions: The Paths of Learning

The second part of a learning trajectory consists of levels of thinking, each more sophisticated than the last, through which children develop on their way to achieving the math goal. That is, the developmental progression describes a typical path that children follow in developing an understanding and skill about that math topic.

> *Humans are born with a fundamental sense of quantity.*
>
> (Geary, 1994, p. 1)

This development of math abilities begins when life begins. As we will see, young children have certain math-like competencies in number, spatial sense, and patterns from birth. However, young children’s ideas and their interpretations of situations are uniquely different from those of adults. For this reason, good early childhood teachers are careful not to assume that children “see” situations, problems, or solutions the way adults do. Instead, good teachers interpret what the child is doing and thinking and attempt to see the situation from the child’s point of view. Similarly, when they interact with the child, these teachers also consider the environment, activities, and their own actions from the child’s point of view so they can help the child develop the next level of thinking. This makes early childhood teaching both demanding and rewarding.

Our learning trajectories provide simple labels and examples for each level of each developmental progression. The “Developmental Progression” column in Table 1.1 describes three main

Table 1.1 Samples from the Learning Trajectory for Counting (the full text for each level, the full learning trajectory, and links to resources are described in Chapter 3)

Age (years)[2]	*Developmental Progression*	*Instructional Activities*
1	**Number Word Sayer: Foundations** *Verbal* No verbal counting. Names some number words with no sequence.	*Number Talk*: Associate number words with quantities and as components of the counting sequence. *Diez Amigos Finger Play* and *Two Little Butterflies Finger Play*: Finger plays like this one are a fun way to teach children about counting and numbers.
1–2	**Chanter** *Verbal* Chants number words in "sing-song" fashion and may run them together. The number words may be indistinguishable from one another. After watching and adult put one to six "food tokens" into an animal puppet, imitates the puppet-feeding with attention to number.	*Verbal counting, songs, finger plays, and more*: Repeated experience with the counting sequence in varied context. This can include songs; finger plays, such as "This Old Man"; counting going up and down stairs; and just verbal counting for the fun of it (how high can you go?)! *Counting with Maracas and More, Use* maracas or other percussion instruments to support the development of number concepts and counting.
3	**Reciter (10)** *Verbal* Verbally counts to ten with some correspondence with objects but may either continue an overly rigid correspondence or exhibit performance errors (e.g., skipping, double counting). "One [points to first], two [points to second], three [starts to point], four [finishes pointing, but is now still pointing to third object], five, ... nine, ten, eleven, twelve, 'firteen,' fifteen ..."	*Count, Clap, and Stomp*: Have all children count from one to ten or an appropriate number, making motions with each count. For example, say, "one" [touch head], "two" [touch shoulders], "three" [touch head], etc.
4	**Counter (Small Numbers)** Accurately counts objects in a line to five and answers the "how many" question with the last number counted, understanding that this represents the total number of objects (the cardinal principle).	*Mr. MixUp*: The puppet Mr. MixUp makes a lot of counting mistakes such as saying the wrong word for "how many" after counting; children help Mr. MixUp by catching his mistake.

levels of thinking in the counting learning trajectory. Under the descriptions are examples of children's thinking and behavior for each level.

Teaching Practices: The Paths of Teaching

The third part of a learning trajectory consists of a set of teaching practices, including educational environments,[3] interactions, and instructional activities, linked to each of the levels of thinking in the developmental progression. These tasks are designed to help children learn the ideas and skills needed to *construct the next level of thinking*. That is, as teachers, we can use these tasks to promote children's growth from the previous level to the goal level. The last column of Table 1.1 provides example instructional activities. (Again, the complete learning trajectory in Chapter 3 includes not only all the developmental levels but also *many more* instructional tasks for each level.)

How do activities help children build each level of thinking? Although teaching and learning resist simple descriptions, we try to embody the *mental* "actions on objects" that enables thinking at a level in children's actions with manipulatives or their bodies (again, Chapter 3 will have more detail; the following are but brief examples). *Count All Day!* in Table 1.1 develops verbal counting with enjoyable activities such as counting in books, songs, finger plays, and clapping or marching up steps. Each allows children to actively produce the verbal counting sequence, with most illustrating the notion of counting-words-as-indicators-of-increasing-quantity (more fingers or higher stairs). The actions are producing number words from an ordered list along with physical action of clapping or marching.

Kitchen Counter's actions include verbal counting, but the computer supports that—the child can focus on the goal of clicking on each object once and only once—an action of attention (like pointing) directed at physical items. The "bite" out of the piece of food *and* error messages as necessary ("You already took a bite out of that one!") to scaffold this one-to-one correspondence activity.

The **Counter (Small Numbers)** level includes a more challenging concept: The last number word reached while counting a set *tells you how many in the set*. Adults find this "obvious," but the concept—*cardinality,* or "how-manyness" in counting—is a significant insight that children must construct. Let's examine the activity *How Many in My Hand?* in more detail (see Figure 1.2). For comparison, first consider that many teachers practice counting with a group by laying out, say, four cubes and asking children to "count with me," leading them in verbal counting as they point to each block, "1, 2, 3, 4." Children do get practice with verbal counting, but the one-to-one correspondence is done by the teacher and may not be noticed by children, and the notion of cardinality is nowhere to be found.

[LT]² ☑ Whole Group ☑ Small Group ! Center ! Computer Center

How Many in a Hand?

Trajectory: Counting
Level: Counter (Small Number)

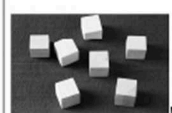
✓ **Quick Description:** Children learn that counting tells how many (that the last number word tells how many in a group. (Adapted from: *Building Blocks*)

Activity

- Secretly put about four inch cubes in one hand and hide it behind your back.
- Tell children you saw the wooden inch cubes and you thought, "I wonder how many I can hold in one hand?"
- Ask children to count aloud with you to find out how many.
- Remove just one of the cubes, and place it where children can see and focus on it. Say "one" with the children.
- Repeat until you have counted and displayed all four cubes. Then show your empty hand.
- Ask children how many cubes there are in all (gesture around them). If they reply with the correct number, agree, gesture around the group of cubes again, and reiterate that, together, you counted 4 cubes.
- Tell children you put the inch cubes in a learning center (or on the tables), and challenge them to find out, during free time, how many *they* can hold in one hand.
- Repeat with a different number of cubes and/or different size objects on subsequent days.

Materials

✓! wooden inch cubes or other similar size objects (an adult should be able to hold 4 or 5 of the objects in one hand)



!

Notes

1. We start with a hidden handful so there is a quantitative question–how many *are there?* –the goal of counting.
2. We lay the cubes out one at a time to help children *use* their ability to *subitize (recognize the number in)* small sets to understand the "cardinality principle" or how-manyness idea of counting. That is, when we say "two" we *see* two and so forth for "three" and "four."

Figure 1.2 The "How Many in My Hand?" activity

In contrast, *How Many in My Hand?* engages children with the concept of cardinality and the cardinality principle in counting (last counting word is "how many") in several ways (see Figure 1.2):

1. Starting by hiding cubes behind the teacher's back immediately makes children curious about cardinality: How many *are* back there?
2. Removing the cubes one at a time evokes children's recognition of small numbers (See Chapter 2). When they count "one" they *see one*, and when they count "two," they *see two*, so the "last number counted" is telling the number they see.
3. The teacher gestures *around* the set and repeats: "Yes, I could hold *four*." Again, reinforcing the notion that the last number word tells how many were counted.
4. The teacher challenges the children to try it themselves, motivating them to figure out how many *they* can hold and making them, not the teacher, the main actors. (They will be motivated—one way or the other—to hold more than four!)

These simple but powerful characteristics of the *How Many in My Hand?* activity help children build the cardinality concept: They learn the mental actions of *unifying* the group (understanding the objects *as* a group) and assigning a number to the group—*quantifying* it.

In summary, learning trajectories describe the goals of learning, the thinking and learning processes of children at various levels along the developmental progression, and the learning activities in which they might engage. People often have several questions about learning trajectories. You may wish to read our responses to those questions that interest you now and return to this section after you have read more about specific learning trajectories in the chapters that follow.

Frequently Asked Questions (FAQ) about Learning Trajectories

Why Use Learning Trajectories? Learning trajectories allow teachers to support the *math of children—the thinking of children as it develops naturally*. Because the trajectories are formed on research of children's natural thinking, we know that all the goals and activities are within the developmental capacities of children. We also know that each level provides a natural *developmental building block* to the next level. We know that the activities provide the *mathematical building blocks* for school success because the research on which they are based typically involves more children who have had the educational advantages that allow them to do well at school.

When are Children "At" a Level? Children are identified to be "at" a certain level when most of their behaviors reflect the thinking—ideas and skills—of that level. Usually, they show a few behaviors from the next and previous levels as they learn. *And we have new empirical evidence that the learning trajectories approach is more effective than other approaches* (Clements, Sarama, Baroody, & Joswick, 2020a; Clements, Sarama, Baroody, Joswick, & Wolfe, 2019).

Can Children Work at More Than One Level at the Same Time? Yes, although most children work mainly at one level (and are starting to learn the next one; of course, if they are tired or distracted, they may operate at a lower level). Levels are not "absolute stages." They are "benchmarks" of complex growth that represent distinct ways of thinking. So, another way to think of them is as a sequence of different *patterns* of thinking and reasoning. Children are continually learning within levels and then moving from one level to the next.

Can Children Jump Ahead? Yes, especially if there are separate "subtrajectories" within a trajectory. For example, we have combined many counting competencies into one "counting" sequence with subtrajectories, including verbal counting and object counting. Many children learn to count to 100 at age 6 after learning to count objects to ten or more; however, some may learn that verbal skill earlier. The subtrajectory of verbal counting skills would still be followed. There is another possibility: Children may learn deeply and thus appear to jump ahead several "levels" after a rich learning experience.

Are all Levels Similar in Nature? Most levels are *levels of thinking*—a distinct period of time of qualitatively distinct ways, or patterns, of thinking. However, a few are merely "levels of attainment," similar to a mark on a wall to show a child's height; that is, a couple signify simply that a child has gained more knowledge. For example, consider reading numerals such as "2" or "9." Children do follow a learning trajectory of first matching, then recognizing, then naming numerals (Wang, Resnick, & Boozer, 1971). However, once they have reached that level, children must learn simply to name (and write) more numerals, which usually does not require deeper or more complex thinking. Thus, some trajectories are more tightly constrained by natural cognitive development than others. Often a critical component of such constraints is the mathematical development in a domain; math is a highly sequential, hierarchical domain in which certain ideas and skills often have to be learned before others.

How are Learning Trajectories Different from just a Scope and Sequence? They are related, of course. But they are *not* lists of everything children need to learn, because they don't cover every single "fact" and they emphasize the "big ideas." Further, they are about children's levels of thinking, not just about the ability to answer a math question. So, for example, a single math problem may be solved *differently* by students at *different* (separable) levels of thinking, even if they all get it right (or wrong!).

Does Every Trajectory Represent Just "One Path"? As mentioned, some trajectories have "subtrajectories." In some cases, the names make this clear. For example, in Comparing and Ordering, some levels are about the "Comparer" levels and others about building a "mental number line." Similarly, the related subtrajectories of "Composition" and "Decomposition" are easy to distinguish. Sometimes, for clarification, subtrajectories are indicated with a note in italics after the title. For example, in Shapes, "Parts" and "Representing" are subtrajectories within the Shapes trajectory. Some children may be further ahead in one subjectory that another.

A more complex question is *whether there is one path every child follows*. Generally, children develop similarly through these broad levels of thinking (they are *not* narrow "lockstep" movements!). However, there are many factors, from cultural to individual, that may account for some children altering that path, usually in small ways (e.g., level 5 before 4).

Frequently Asked Questions (FAQ) about Using Learning Trajectories

How Do These Developmental Levels Support Teaching and Learning? The levels help teachers (as well as curriculum developers) understand children's thinking; the ability to create, modify, or sequence activities. *Teachers who understand learning trajectories (especially the developmental levels that are at their foundation) are more effective, efficient, and fun for everyone.*

Through planned teaching and also by encouraging informal, incidental math, teachers help children learn *at an appropriate and deep level*.

There are Ages in the Charts. Should I Plan to Help Children Develop Just the Levels that Correspond to my Children's Ages? No! The ages in the table are typical ages at which children develop these ideas. But these are rough guides only—children differ widely. Furthermore, the children achieve *much later levels* with high-quality education. So, these are approximate "starting levels," not goals. Children who are provided high-quality math experiences are capable of developing to levels one or more years beyond their peers.

Are the Instructional Tasks the Only Way to Teach Children to Achieve Higher Levels of Thinking? No, there are many ways. In some cases, however, there is some research evidence that these are especially effective ways. In other cases, they are simply illustrations of the kind of activity that would be appropriate to reach that level of thinking. Further, teachers need to use a variety of pedagogical strategies in teaching the content, presenting the tasks, guiding children in completing them, and so forth.

Are Learning Trajectories Consistent with Teaching the Common Core? Unfortunately, some people have interpreted that "teaching the Common Core" means only teaching each standard directly and then moving on. But learning is not an all-or-nothing acquisition of knowledge or skills (Sarama & Clements, 2009c; Sophian, 2013). The Common Core goals are benchmarks, but good curricula and teaching always work *up to those goals* and weave the learning opportunities throughout children's lives. They learn the ideas at higher levels of sophistication and generality. Finally, *when we wrote the Common Core, we started by writing learning trajectories—at least the goals and developmental progressions*. Thus, learning trajectories are at the *core* of the Common Core. And learning trajectories are *not* based on the idea to "directly teach it once and drop it."

Before we leave the Common Core, we note that misconceptions and misinformation about the CCSSM standards abound, *especially* the erroneous idea that they are "developmentally appropriate" for the youngest children. We know if children have opportunities to learn, they can meet and exceed all those standards. If you need accurate information about the CCSSM, please see our many articles on the topic (Clements, Fuson, & Sarama, 2017a; 2017b, 2019; Fuson, Clements, & Sarama, 2015).

Other Critical Goals: Strategies, Reasoning, Creativity, and a Productive Disposition

Learning trajectories are organized around topics, but they include far more than concepts, facts, and skills. Processes, or math practices, and attitudes are important in every one. Chapter 13 focuses on general processes, such as problem solving and reasoning. But these and other general processes are also an integral part of every learning trajectory. Also, specific processes are involved in every learning trajectory. For example, the process of composition—putting together and taking apart—is fundamental to both number and arithmetic (e.g., adding and subtracting) and geometry (shape composition).

Finally, other general educational goals must never be neglected. The "habits of mind" mentioned in the box include curiosity, imagination, inventiveness, risk-taking, creativity, and persistence. These are some of the components of the essential goal of *productive disposition*.

Children need to view math as sensible, useful, and worthwhile and view themselves as capable of thinking mathematically. Children should also come to appreciate the beauty and creativity that is at the heart of math. Remember Albert Einstein's quote at the beginning of the preface: "Mathematics is, in its way, the poetry of logical ideas."

All these should be involved in a high-quality early childhood math program. These goals are included in the suggestions for teaching throughout this book. Further, Chapters 14, 15, and 16 discuss how to achieve these goals. These chapters discuss different learning and teaching contexts, including early childhood school settings and education, equity issues, affect, and instructional strategies.

> *As important as mathematical content are general mathematical processes such as problem solving, reasoning and proof, communication, connections, and representation; specific mathematical processes such as organizing information, patterning, and composing, and habits of mind such as curiosity, imagination, inventiveness, persistence, willingness to experiment, and sensitivity to patterns. All should be involved in a high-quality early childhood mathematics program.*
>
> (Clements, 2004, p. 57)

Learning Trajectories and the "Building Blocks" Project

The "Building Blocks" project was funded by the National Science Foundation (NSF)[4] to develop pre-kindergarten (pre-K) to Grade 2 software-enhanced, math curricula. Building Blocks was designed to enable all young children to build math concepts, skills, and processes. The name "Building Blocks" has three meanings (see Figure 1.3). First, our goals are to help children develop the main *mathematical building blocks*—that is, the *big ideas* described previously. Second is the related goal to develop *cognitive building blocks*: general cognitive and metacognitive (higher-order) processes such as moving or combining shapes to higher-order thinking processes such as self-regulation. The third is the most straightforward: Children should be using building blocks for many purposes, but one of them is for learning math.

Based on theory and research on early childhood learning and teaching (Bowman, Donovan, & Burns, 2001; Clements, 2001), we determined that Building Blocks' basic approach would be *finding the mathematics in, and developing mathematics from, children's activity.* To do so, all aspects of the Building Blocks project are based on learning trajectories. Many of the examples of learning trajectories stemmed from our work developing, field-testing, and evaluating curricula from that project. Praised on the cover of *The New York Times* and the *Wall Street Journal* and validated by the "What Works Clearinghouse," this project was the genesis of this book as well as the web-based tool that we turn to next.

> *The overriding premise of our work is that throughout the grades from pre-K through 8 all students can and should be mathematically proficient. [p. 10]*

Mathematical proficiency ... has five strands:

1 *conceptual understanding—comprehension of mathematical concepts, operations, and relations*
2 *procedural fluency—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately*
3 *strategic competence—ability to formulate, represent, and solve mathematical problems*
4 *adaptive reasoning—capacity for logical thought, reflection, explanation, and justification*
5 *productive disposition—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.*

(Kilpatrick, Swafford, & Findell, 2001, p. 5)

The Learning and Teaching with Learning Trajectories Tool

To help teachers understand and teach the Building Blocks curriculum, we created an Internet site that featured descriptions and videos of children's thinking and instructional activities that developed it (e.g., see Sarama & Clements, 2003). Teachers found it so useful that we created a new site, the *Learning and Teaching with Learning Trajectories*[5]

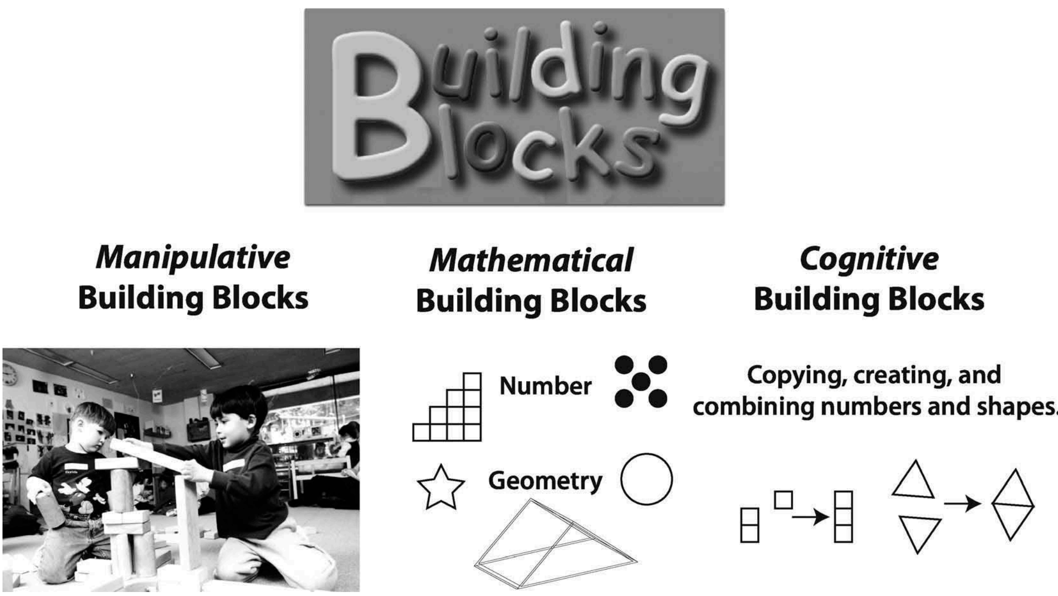


Figure 1.3 The Building Blocks project was named because we wanted to use manipulatives like children's building blocks (on and off the computer) to help children develop mathematical and cognitive building blocks—the foundations for later learning (see http://buildingblocksmath.org)

tool at www.LearningTrajectories.org. There you can see videos of children's thinking at *each level* of each topic (learning trajectory) as in Figure 1.4, as well as *classroom* and home videos of teachers and caregivers helping children *learn* that level of thinking. Each instructional activity has PDF files, that you can print out and use, fully describing the activity, along with materials (e.g., shape printouts) to accompany the activities, *and* links and notes on how to make sure *all* children, including children with disabilities, can fully engage in each activity. [LT]^2 also features an extensive Resource section with videos, articles, and links on teaching and on particular topics and issues of teaching (e.g., dual language learners).

(a)

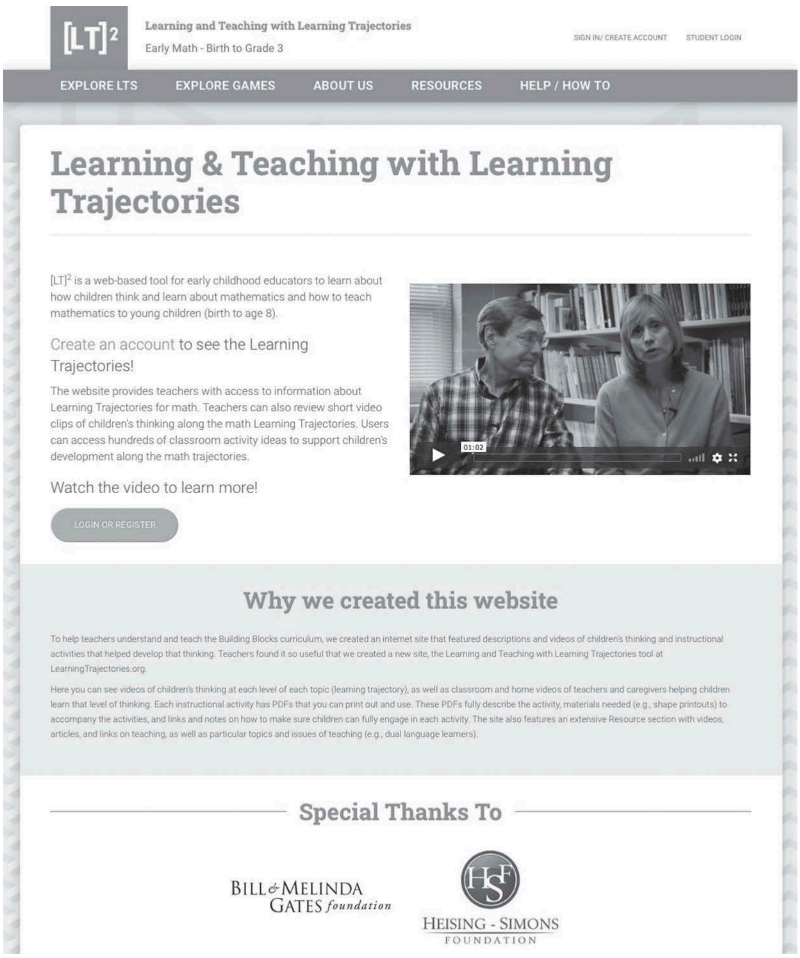
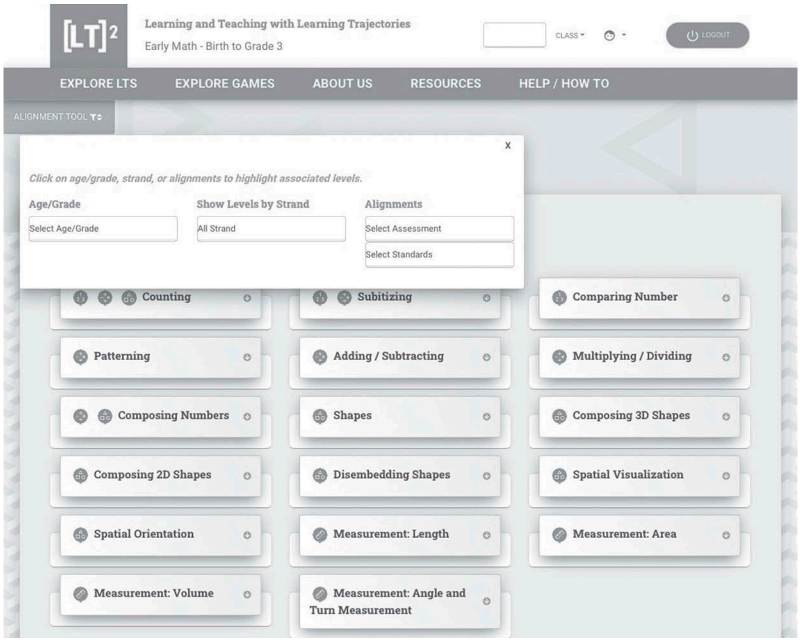


Figure 1.4 The *Learning and Teaching with Learning Trajectories* (LTLT, OR [LT]^2) tool at www.LearningTrajectories.org. (a) presents the home screen of [LT]^2

(b)



(c)

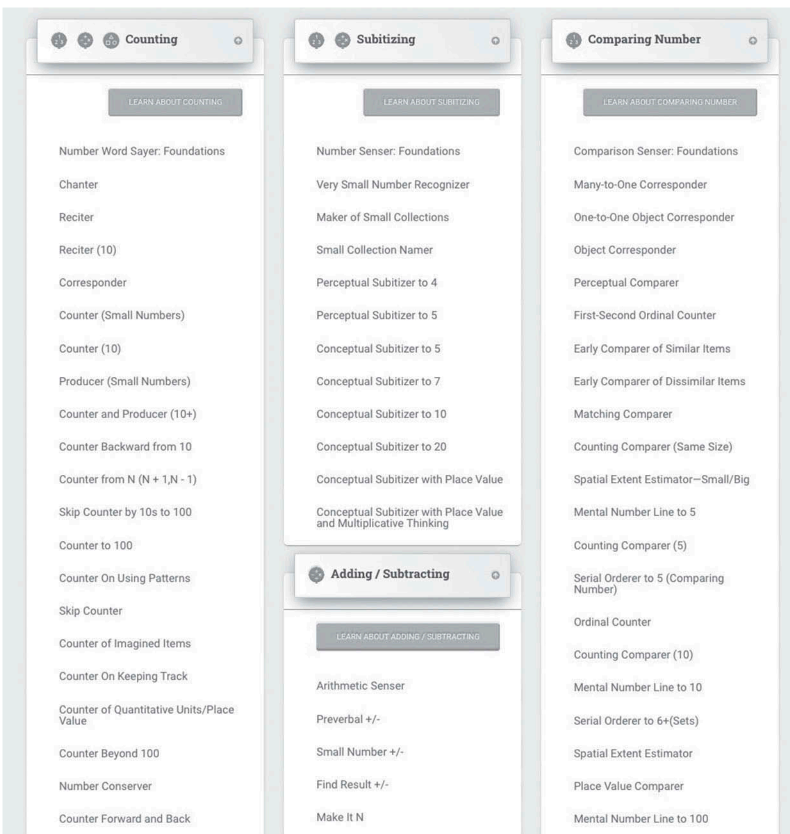


Figure 1.4 (continued) (b) [LT]^2 includes full research–validated learning trajectories for all topics in early math and alignments with many national and state standards and assessments. (c) For each topic, once "opened" a "Learn about …" section teaches users about the goal, and a full list of levels of the developmental progression

(d)

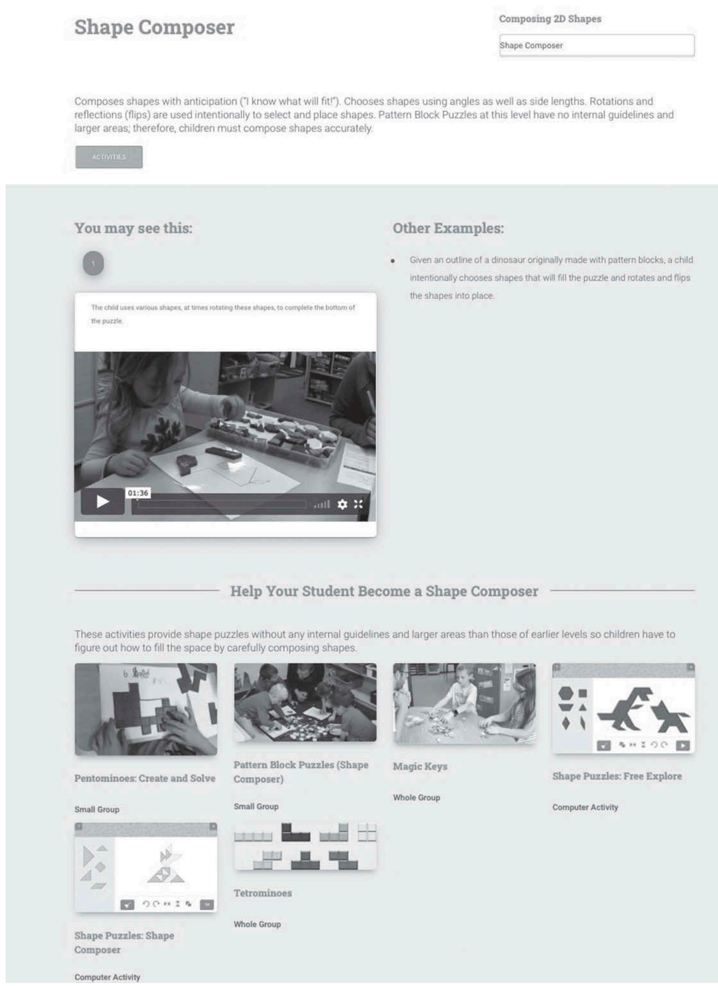


Figure 1.4 (continued) (d) For each level, [LT]^2 provides a definition, one or more videos, and descriptions of children's thinking for each level of each developmental progression, and then instructional activities that teach that level

We encourage you, as you read about each level, to go to [LT]^2 *and see videos of children that illustrate that level of thinking and then see (and use!—when appropriate) videos and other resources for instructional activities that help children build that level of thinking.*

Final Words

Against this background, let us explore the learning trajectories in Chapters 2 through 12. Chapter 2 begins with the critical topic of *number*. When do children first understand number? *How* do they do it? How can we help children's initial ideas develop? Throughout, we emphasize math processes, or practices, and attitudes. Further, the last few chapters provide guidance regarding understanding children, communities, and cultures, and tools such as effective teaching strategies. You may want to at least skim Chapter 13 before reading the following chapters on learning trajectories.

(e)

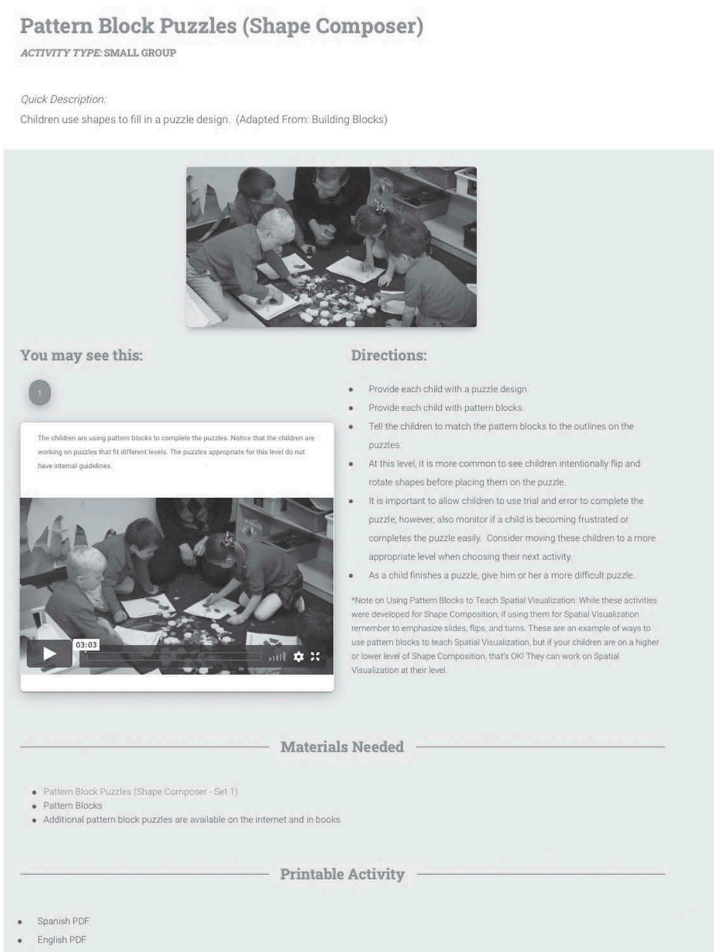


Figure 1.4 (continued) (e) For each of instructional activity, [LT]^2 provides directions, videos, and a set of downloadable, carefully formatted PDF files of the activity as well as materials for the activities in English and Spanish

Remember, we encourage you to go to [LT]^2 *at LearningTrajectories.org and actually* ***see*** *children at each level of development and the activities that helped them develop each level.*

Before we move forward, let's review the reasons early math is surprisingly important.

The Surprising Importance of Early Math: A Summary

1 *Math is important, but math teaching and learning has not improved in the USA, including in the youngest children. Better early math for all helps everyone: strong math skills = social progress.*
2 *Early math learning, from birth, is critical for all future learning … and living. Early math promotes math, but also social, emotional, literacy, and general brain development. There is much to gain and nothing to lose from high-quality early mathematics.*

3 *All children deserve fair opportunities to learn. We need powerful "charging stations" for math in all communities. Math should be purposeful, relevant, and fun for all children, not passive, irrelevant, and tedious for some.*
4 *From their first years, children have boundless interest in and curiosity for math … and the ability to learn to think like mathematicians. Math is a language best learned early. And young children love to think mathematically, to see the world through a mathematical lens in new and powerful ways.*
5 *Teachers and families enjoy all that high-quality math brings to their children. And research provides the tools math makes math easier, more effective, and more enjoyable.*

(f)

Figure 1.4 (continued) (f) [LT]^2 provides a variety of resources for all users, including videos about various issues and topics for a variety of users, guides for professional developers, and so forth

Notes

1 One of the authors, Douglas Clements, was a member of the NMAP and co-author of the report, which can be found at www.ed.gov/about/bdscomm/list/mathpanel/.

2 The ages in the tables are typical ages at which children develop these ideas. However, children vary widely and just as important, *with high-quality education*, children achieve much later levels.

3 Environments and interactions are important—for infants and toddlers, foundations for math are embedded in rich materials and structures in the environments and interesting, everyday interactions with adults and peers. This continues throughout early childhood education but the role of intentional activities increases as developmentally appropriate—engaging, meaningful, challenging-but-achievable!

4 The "Building Blocks—Foundations for Mathematical Thinking, Prekindergarten to Grade 2: Research-based Materials Development" project was funded by the NSF (award no. ESI-9730804; granted to D. H. Clements and J. Sarama) to create and evaluate math curricula for young children based on a theoretically sound research and development framework. We describe the framework and research in detail in Chapter 15. For the purposes of full disclosure, note that we have subsequently made this curriculum available through a publisher and thus receive royalties. All research was conducted with independent assessors and evaluators.

5 Funded by the Heising-Simons Foundation and the Bill and Melinda Gates Foundation, *Learning and Teaching with Learning Trajectories* is also known by its initials, LTLT, or, therefore, as $[LT]^2$ (one of those "math jokes" almost totally devoid of actual humor).

NOTES

Preface

1 Like many acronyms, TRIAD *almost* works … we jokingly ask people to accept the "silent p" in "professional development."

Chapter 1

1 One of the authors, Douglas Clements, was a member of the NMAP and co-author of the report, which can be found at www.ed.gov/about/bdscomm/list/mathpanel/.
2 The ages in the tables are typical ages at which children develop these ideas. However, children vary widely and just as important, *with high-quality education*, children achieve much later levels.
3 Environments and interactions are important—for infants and toddlers, foundations for math are embedded in rich materials and structures in the environments and interesting, everyday interactions with adults and peers. This continues throughout early childhood education but the role of intentional activities increases as developmentally appropriate—engaging, meaningful, challenging-but-achievable!
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Chapter 2

1 "Number sense" includes a large number of competencies, including composing and decomposing numbers, recognizing the relative magnitude of numbers, dealing with the absolute magnitude of numbers, using benchmarks, linking representations, understanding the effects of arithmetic operations, inventing strategies, estimating, and possessing a disposition toward making sense of numbers (Sowder, 1992b).

2 Later grades use subitizing in many ways, such as in supporting the development of counting concepts and skills and solving arithmetic problems. These goals will be highlighted in subsequent chapters.
3 Funded by the Heising-Simons Foundation and the Bill and Melinda Gates Foundation, *Learning and Teaching with Learning Trajectories* is also known by its initials, LTLT, or, therefore, as $[LT]^2$ (one of those "math jokes" almost totally devoid of actual humor).
4 The ages in the table are typical ages at which children develop these ideas. However, children vary widely and just as important, *with high-quality education*, children achieve much later levels. See p. 9 in Chapter 1.

Chapter 3

1 Research confirms recommended practice: Math education should start from the *earliest* years (Hojnoski, Caskie, & Miller Young, 2018).

Chapter 4

1 See Chapter 6 for much more information about place value.

Chapter 5

1 Several important and complex issues regarding manipulatives are discussed at length in Chapter 16.

Chapter 6

1 We use the term "combination" instead of the common term "fact" for two reasons. First, "facts" implies they are verbal knowledge to be memorized by rote. We believe they are number relationships that are understood in a variety of ways that must be constructed by the child. Second, in contrast, "combination" implies that two numbers are decomposed to make another number, and that there are many related combinations (3 + 2 = 5; 2 + 3 = 5; 5 = 2 + 3; 5 − 2 = 3, etc.).

Chapter 7

1 *Motion*: slide or turn. *Direction*: for slides, which way it is headed; for turns, clockwise or counterclockwise. *Amount*: for slides, how far, or turns, how much of a turn (in degrees).

Chapter 8

1 Relax and enjoy. Most of *us* were badly taught math, and especially geometry (Shahbari, 2017). Take your time and it *will* shape up for you!

Chapter 9

1 This deflates the argument, "I don't want my children to have to learn math, I want them to play with blocks!" does it not?

2 Although commonly associated with computers, technology is at many levels, from the lower (wheels, hammers … blocks) to digital technologies.

Chapter 10

1 Compared to discrete quantities, which can be counted by whole numbers (exactly "4 dogs are here"), continuous quantities are those where there is no limit in how small the parts are into which it can be divided ("together the dogs weigh about 117.3 kg"). Scientific measurement with tools can give us only an approximate measure—to the nearest kilogram or pound, or the nearest 1/100th of a kg, but never an exact number.

Chapter 11

1 We include a brief discussion of non-geometric measurement—time and weight—toward the end of this chapter.

Chapter 12

1 For example, patterns represented by two attributes of change (shape and color) are easier than those represented by just one (e.g., orientation). Further, this may be more difficult for some children or populations of children (Warren, Miller, & Cooper, 2012).

2 Fixing a pattern is easier than the other if only one item is missing but may be more difficult if more than one is missing.

Chapter 13

1 Most of the information regarding teaching problem solving is integrated within the content chapters.

Chapter 14

1 Children who can catch up, especially with high-quality instruction, may be developmentally delayed, but not disabled. The Response to Intervention (RTI) model includes this basic idea: If children are behind because of a lack of high-quality experiences and education, *they* have no "mathematical difficulties"; their environment is to blame and must be improved.

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