Critical load of unbraced steel frames with classical and semi-rigid joints

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ABSTRACT: The paper presents the stability issue of unbraced steel frames with special attention paid to the problem of determining the value of critical load in these frames.

The paper discusses ways of critical load determination in frames with the use of selected methods of stability analysis. The presented analysis procedures were used in calculations of certain steel frames.

The issue presented in the paper is important from the practical point of view, and therefore, it should be taken into account in the design process of such frames.

1 INTRODUCTION

Stability is one of the fundamental issues, which has to be taken into account in design of steel frames (Bródka et al. 2004, Chan et al. 1999, Chen et al. 1996, Pałkowski 2016). This issue is particularly important in the case of unbraced (sway) frames, especially in instance of unbraced frames with semi-rigid joints, which are known to be highly susceptible to stability loss.

During analyzing of unbraced frames stability, it is easy to see that the first (basic) buckling mode of the frame stands out first of all by horizontal displacements of the frame nodes. The sway-buckling mode is characteristic not only for the unbraced portal frames, but also in the case of multi-storey and multi-aisle frames, regardless of the location of the load acting on the individual frame nodes.

On the basis of performed analysis, the author of the work stated that the typical unbraced frame loses its stability under the influence of a certain, total load (a sum of loads acting on individual frame nodes), the value of which, approximately, is independent from the load distribution in nodes. This issue can be presented on a simple example of certain unbraced frame, which is loaded by the forces P_i applied to the nodes (Figure 1).

The presented frame was subjected to the stability analysis (using the eigenvalues method of stiffness matrix), with taking into account 9 different load cases, which are listed in Table 1. Based on this analysis, the critical load multipliers λ_{cr} and the total values of the critical nodal forces P_{CR} were obtained for all cases (Table 1).

As it is easy to see, despite of the significant differences in the number and location of the forces P_i , the similar value of the total critical load P_{CR} of the frame was obtained for each considered case. The largest disagreement appears between the case VII and IX and amounts to less than 10%, while the differences in the results between other cases does not exceed 5%.

The slight variations in the results between the presented load cases show that the resistance to the loss of stability of the frame largely depends on its lateral stiffness. This is evidenced by the way the analyzed frame loses its stability – for the all 9 cases, the bucking modes are characterized by horizontal displacements of the frame nodes.

Figure 1. Analyzed frame.

Table 1. The list of load cases accepted for the analysis and the calculation results.

Load case	P_1 [kN]	P_2 [kN]	P_3 [kN]	P_4 [kN]	λ_{cr}	$P_{CR} = \lambda_{cr} \Sigma P_i$ [kN]
II	100	100	100	0	16.86	5058
III	100	100	0	100	17.16	5147
IV	100	100	0	0	25.25	5050
V	0	100	100	0	24.59	4917
VI	100	0	100	0	25.19	5037
VII	100	0	0	100	25.92	5183
VIII	100	0	0	0	49.36	4936
IX	0	100	0	0	46.89	4689

2 THE APPROXIMATE METHOD OF DETERMINING THE CRITICAL LOAD OF UNBRACED FRAMES

The frames property presented in the previous section, manifested by slight differences in the total P_{CR} value of the frame for different load cases can be used in practical calculations of steel frames stability. For this purpose, the stability analysis, e.g., with the use of numerical methods, can be performed for the first (basic) load case, while calculations of the critical load multiplayer for the rest of load cases (or the load combinations) can be obtained according to the relation:

$$\lambda_{cr} = \frac{\sum P_{cri}}{\sum P_i} = \frac{P_{CR}}{\sum P_i} \quad (1)$$

where: P_{CR} is the total value of the critical load determined for the first load case (basic combination), while ΣP_i is a sum of the loads assigned to the next considered load combination.

In the case, where the considered static scheme includes different types of loads (e.g., linear loads, surface loads, concentrated moments, etc.) the above condition can be replaced with the use of the equation:

$$\lambda_{cr} = \frac{\sum N_{cri}}{\sum N_i} = \frac{N_{CR}}{\sum N_i} \quad (2)$$

where $\Sigma N_{cr,i} = N_{CR}$ refers to the combined value of the critical forces, which was determined for the basic load combination, whereas ΣN_i determines the sum of the normal forces in columns, in currently considered combination.

The critical load multiplier λ_{cr} calculated in this way can be used for the approximate determination of a state of critical forces in loading cases or combinations, for which the results of static analysis are known.

3 PARAMETRIC ANALYSIS

In order to check the accuracy of the critical load determination with the use of the presented in section 2 the way, the parametric analysis of a four-aisle, four-storey frame presented in Figure 2 was performed, with taking into account 10 different load cases.

Data: $EI_c = 15000 \text{ kNm}^2$, $EI_b = 30000 \text{ kNm}^2$, $l = 6 \text{ m}$, $h = 4 \text{ m}$, $P_i = 100 \text{ kN}$

First, the numerical analysis was performed in the SOFISTIK program (Sofistik, 2018) according to the eigenvalue method for all the load cases. Then, the critical load multipliers were determined according to the proposed in section 2 algorithm for 9 load cases (from II to X), assuming the result of numerical calculations for the first case as the base value, with the use of which calculations according to formula (1) were performed. The results of the calculations are presented in Table 2.

On the basis of buckling analysis of the presented frame, it can be seen that the largest differences between the P_{CR} values of the considered load cases do not exceed 10%. These differences are not significant, if one takes into account the load distribution of the static scheme in the individual cases, both in terms of the forces number and the locations of their application.

Moreover, presented in Table 2 the results indicate that the simplified calculating method of the critical load multiplier proposed in the paper allows to estimate its value with accuracy of 10%.

It is worth noting that the analyzed example does not present the load model with the permanent load occurring in the real structure (in the analyzed example of the frame, such load would occur in each node). Adoption of the permanent load in the analysis causes that the differences in the values of the total critical load P_{CR} between the individual cases are much smaller. The results of such calculations of the frame for the two selected load cases are presented in Table 3.

Figure 2. Analyzed four-aisle, four-storey frame.

Table 2. The list of load cases accepted for the analysis and the calculation results.

Load case	Scheme of nodal forces					λ_{cr} / P_{CR}	λ_{cr}	Difference
	node number), $P = 100$ kN					$/ P$		
	axis A	axis B	axis C	axis D	axis E	num. calc.	eq. (1)	%
I (basic)	4) P 3) P 2) P 1) P	8) P 7) P 6) P 5) P	12) P 11) P 10) P 9) P	16) P 15) P 14) P 13) P	20) P 19) P 18) P 17) P	10.34/206.86	-	-
II	4) P	8) P	12) P	16) P	20) P	41.22/206.10	41.37	0.4
III	2) P	6) P	10) P	14) P	18) P	41.23/206.15	41.37	0.3
IV	1) P	5) P	9) P	13) P	17) P	41.71/208.57	41.37	-0.8
V	4) P 3) P 2) P 1) P	8) - 7) - 6) - 5) -	12) P 11) P 10) P 9) P	16) - 15) - 14) - 13) -	20) P 19) P 18) P 17) P	17.26/207.14	17.24	0.1
VI	4) - 3) - 2) - 1) -	8) P 7) P 6) P 5) P	12) - 11) - 10) - 9) -	16) P 15) P 14) P 13) P	20) - 19) - 18) - 17) -	25.00/200.00	25.86	3.4
VII	4) P 3) P 2) P 1) P	8) - 7) - 6) - 5) -	12) - 11) - 10) - 9) -	16) - 15) - 14) - 13) -	20) - 19) - 18) - 17) -	47.84/191.36	51.72	8.1
VIII	4) - 3) - 2) - 1) -	8) P 7) P 6) P 5) P	12) - 11) - 10) - 9) -	16) - 15) - 14) - 13) -	20) - 19) - 18) - 17) -	47.17/188.66	51.72	9.6
IX	4) P	8) -	12) -	16) -	20) -	191.42/191.42	206.86	8.1
X	1) -	5) -	9) -	13) -	17) P	191.35/191.35	206.86	8.1

Table 3. The list of load cases accepted for the analysis and the calculation results.

Load case	Scheme of nodal forces					λ_{cr} / P_{CR}	λ_{cr}	Difference
	node number), $P = 100$ kN					$/ P$		
	axis A	axis B	axis C	axis D	axis E	num. calc.	eq. (1)	%
XI	4) G 3) G 2) G 1) G	8) G+P 7) G+P 6) G+P 5) G+P	12) G 11) G 10) G 9) G	16) G 15) G 14) G 13) G	20) G 19) G 18) G 17) G	8.58/205.92-	8.61	0.3
XII	4) G 3) G 2) G 1) G	8) G 7) G 6) G 5) G	12) G 11) G 10) G 9) G	16) G 15) G 14) G 13) G	20) G 19) G 18) G 17) G+P	9.86/207.14	9.85	0.1

Comparing the load case I (Table 2) and the cases XI and XII (Table 3), it can be clearly seen that the differences in the P_{CR} values are very small. Also, the estimation of the parameter λ_{cr} using formula (1) is very accurate.

On the basis of the performed analysis, it can be assumed that for the sway frames, even with significantly different load cases, the total value of the critical load will be approximately constant in value, but characteristic for a given frame.

Figure 3. Analyzed frame with semi-rigid joints.

4 NUMERICAL EXAMPLE OF A FRAME WITH SEMI-RIGID JOINTS

The validity of the presented regularity in frames with semi-rigid joints was checked on an example of three-aisle, three-storey steel frame, shown in Figure 3.

The calculations were performed in the Sofistik program, according to the eigenvalues method for 10 load cases. Determination of the critical load multipliers was also performed according to the proposed in section 2 algorithm for 8 load cases (from II to IX), accepting the result of the numerical calculations for the I case as a basic value. The results of the preformed calculation were listed in Table 4.

Data: steel grade S355, steel modulus of elasticity $E = 210$ GPa.

It was assumed that two types of loads act on the load-bearing structure: g – permanent load with the value of 25 kN/m and 35 kN/m, and q – variable load with the value of 35 kN/m (Figure 3).

On the basis of the static analysis and strength calculations of that frame, the elements cross-sections were selected. Then, based on recommendations of the standard (EN 1993-1-8, 2005), the frame joints were designed in the form of single- and double-side end-plate bolted joints. The column bases in the form of anchored horizontal plates were also calculated.

Finally, in order to carry out the buckling analysis, the average stiffnesses of the internal joints (beam-to-column joints) equal $S_{j,A} = 60 \times 10^3$ kNm/rad, as well as the stiffnesses of the external joints (column bases) equal $S_{j,B} = 15 \times 10^3$ kNm/rad was adopted in the model. Appropriate stability calculations were performed, as of the parametric analysis, in the SOFISTIK program (Sofistik, 2018), and also according to the algorithm presented in section 2. The calculation results are presented in Table 4.

The obtained results clearly demonstrate that the total values of the critical loads P_{CR} , which were determined for the presented load cases reach very similar values. The largest differences between obtained values do not exceed 2%.

Furthermore, the presented calculation results show very good agreement of the parameters λ_{cr} , which were calculated on the basis of numerical method and according to proposed in section 2, simplified way of determining the critical load.

Table 4. The list of load cases accepted for the analysis and the calculation results.

Load case	Load $g = 35, g = 25, q = 35$				λ_{cr}	$P_{CR} = \lambda_{cr} \Sigma (g_i + q_i) l_i$	λ_{cr}	Difference
	kN/m				kN			
	storey	aisle I	aisle II	aisle III	numerical analysis		eq. (1)	%
I (basic)	3	g	g	g	14.56	24.9×10^3 kN	-	-
	2	g	g	g				
	1	g	g	g				
II	3	g	g	g	10.82	25.3×10^3 kN	10.64	-1.7
	2	g	g	g				
	1	$g+q$	$g+q$	$g+q$				
III	3	g	g	g	10.52	24.6×10^3 kN	10.64	1.1
	2	$g+q$	$g+q$	$g+q$				
	1	g	g	g				
IV	3	g	g	g	8.44	25.1×10^3 kN	8.38	-0.7
	2	$g+q$	$g+q$	$g+q$				
	1	$g+q$	$g+q$	$g+q$				
V	3	g	g	g	11.70	24.9×10^3 kN	11.69	-0.1
	2	g	$g+q$	g				
	1	g	$g+q$	g				
VI	3	g	g	g	11.73	25.0×10^3 kN	11.69	-0.3
	2	$g+q$	g	g				
	1	$g+q$	g	g				
VII	3	g	g	g	9.80	25.0×10^3 kN	9.77	-0.3
	2	$g+q$	$g+q$	g				
	1	$g+q$	$g+q$	g				

5 SUMMARY AND FINAL REMARKS

The paper presents the issue of critical load determination of the unbraced steel frames. On the basis of the stability analysis, it was noticed that structures of this type generally loses their stability under the influence of certain, total value of load, which essentially is independent from the assumed load scheme. The occurrence of this regularity was confirmed on the example of the single-storey, multi-aisle frame.

On the basis of the noticed property of unbraced frames, in the section 2 of the work, the algorithm of approximate determination of the critical load value was proposed. The practical way of calculating and the accuracy of the results obtained according to this algorithm is shown on the example of the four-aisle, four-storey frame.

A detailed stability analysis on the example of the real steel frame with the internal and external semi-rigid joints was also performed.

The results comparisons of the presented in the paper examples, which were obtained with the use of the software and the equation (1) prove, that the proposed in the paper algorithm of the critical load determination is very useful in practical calculations of unbraced frames.

Although the above-presented conclusions refer directly to the examples presented in the paper, it should be assumed that they are more of general nature and relate mostly to the unbraced frames used in practice.

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