Robust optimisation of a single-layer lattice dome

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ABSTRACT: The formulation of deterministic optimization in no way takes into account the randomness of the design variables [Chybiński, M., Garstecki, A. (2017), Czubacki, R., Lewiński T. (2020)]. Optimum structures are particularly sensitive to parameter imperfections. Optimal solutions located on the border of the acceptable area may relatively easily turn out to be completely useless if the parameter values differ from the assumed nominal values. It seems natural to extend the formulation of deterministic optimization, which takes into account the uncertainty of parameter values. Robust optimization proposed in the paper offers such possibilities. In the paper, robust optimisation is discussed on the example of a single-layer lattice covering. After dimensioning the individual groups of bars, the safety level of the structure was assessed by determining the reliability index and failure probability. The structure under analysis is susceptible to stability failure resulting from the condition of the node snap-through. On this basis, a displacement limit function was adopted, which refers to the maximum displacement value at the instant of the node snap-through. Next, two methods of structure optimisation (deterministic and robust), based on analogous constraints and objective function, were compared. The comparison of both methods ends with a reassessment of the safety level of the structure. As a result of robust optimization, a structure with a slightly larger mass was obtained. The difference in the weight of the structure in the case of deterministic and immunity optimization did not exceed 2%. However, the reliability index, which measures the safety of the structure, has increased significantly. At the expense of a slight increase in weight, we obtained a structure that is more reliable and resistant to the dispersion of parameters characterizing the structure's operation.

1 INTRODUCTION

The notion 'robust design' covers broadly understood methodology of designing structures, devices and also manufacturing processes, which aims to find a solution showing the highest resistance to changes in parameters. At the same time, high functionality of the systems designed is to be maintained. Reduction in the variance of quantities that characterize the structure performance can be achieved by either limiting, or eliminating the scatter of the values of input parameters. In practice, such an approach generates too high costs, making it unviable. A far better method involves a choice of selected design variables made in such a way that a design resistant to parameter variances is obtained, but variances in the structure random parameters do not change. Robust optimisation falls into the methodology category specified above.

Figure 1. The concept of robust optimisation for a single design variable.

The concept of robust optimisation, shown in Figure 1, is explained on the example of a problem with one design variable X . When a constant variance of the X random variable is assumed, it can be observed that for the solution $\mu_X^{(2)}$, the objective function variance is decidedly lower than for solution $\mu_X^{(1)}$. In the latter case, the reduction in the average value of the objective function is accompanied by a significant increase in variance.

Robust optimisation represents non-deterministic optimisation formulations. That is due to the fact this approach takes into account the impact of the random nature of the structure parameters on the scatter of the structure response, which usually leads to an increase in the structure reliability. A typical objective function in robust optimisation problem usually includes terms of the mean value, and of variance of the structure response function. Constraints can be deterministic, or can be expressed by the first two statistical moments of response. Contrary to other optimisation types (e.g. reliability optimisation), imprecise determination of types of probability distributions is not of primary importance. The values of the first statistical moments of the structure responses depend primarily on the first moments of random variables. When relevant data are not available, a uniform or normal distribution of variables is often assumed [Błachowski et al. (2020), Tazowski et al. (2019)]. Interesting modern optimization methods using e.g. genetic algorithms are presented in the paper [Miller, Ziemiański (2020)]. The paper [Rozvany et al (2014)] deals with so-called “optimal plastic design” of trusses with multiple loads, which is based on ultimate load limit principles and requires only static admissibility of the solution.

2 PROBLEM FORMULATION

The aim of robust optimisation is a simultaneous minimization of both the average value and random variation of the objective function. The standard deviation is most often used to describe the scatter. In the formulation below, a two-element vector of the optimisation criteria is found, namely: mean value $E[f(\mathbf{d}, \mathbf{X}, \mathbf{P})]$, and standard deviation $\sigma[f(\mathbf{d}, \mathbf{X}, \mathbf{P})]$, which accounts for multi-criteria optimisation problem formulation. The task of robust optimisation can be formulated as follows:

Find values of variables:

$$\mathbf{d}, \mu_X \quad (1)$$

that minimize:

$$\{E[f(\mathbf{d}, \mathbf{X}, \mathbf{P})], \sigma[f(\mathbf{d}, \mathbf{X}, \mathbf{P})]\} \quad (2)$$

with the constraints:

$$E[g_i(\mathbf{d}, \mathbf{X}, \mathbf{P})] - \tilde{\beta}_i \sigma[g_i(\mathbf{d}, \mathbf{X}, \mathbf{P})] \geq 0, \quad i = 1, \dots, k_g, \quad (3)$$

$$\sigma[c_k(\mathbf{d}, \mathbf{X}, \mathbf{P})] \leq \sigma_k'', \quad k = 1, \dots, k_c, \quad (4)$$

$$d_j^l \leq d_j \leq d_j'', \quad j = 1, \dots, n_d, \quad (5)$$

$$\mu_{x_r}^l \leq \mu_{x_r} \leq \mu_{x_r}^u, \quad r = 1, \dots, n_x. \quad (6)$$

where: $\mathbf{d}$ – deterministic design variables, $\mathbf{X}, \mathbf{P}$ – vectors of random variables with expected values of μ_x, μ_p , f – objective function, g_i – functions of constraints, c_k – functions, the standard deviations of which must not exceed the allowable values σ_k'' , $\beta_i > 0$ – coefficients corresponding to the constraints $g_i \geq 0$ which represent the safety margin with which these constraints must be met.

The concept of constraints is shown in Figure 2. While solving the problem of robust optimisation, it is sought that, at the optimum point, the mean values of the constraint functions g_i , $i=1, \dots, k_g$, should be shifted away from zero by at least β_i standard deviations. A very popular way to solve multi-criteria optimisation tasks is to employ the problem scalarization method. In the method, a linear combination of criteria is used as an objective function. Consequently, the problem (7) – (12) can be modified to obtain the following scalar optimisation problem:

Find values of variables:

$$\mathbf{d}, \mu_x \quad (7)$$

that minimize:

$$\tilde{f} = \frac{1 - \alpha}{\mu^*} \mathbf{E}[f(\mathbf{d}, \mathbf{X}, \mathbf{P})] + \frac{\alpha}{\sigma^*} \sigma[f(\mathbf{d}, \mathbf{X}, \mathbf{P})] \quad (8)$$

with the constraints:

$$\mathbf{E}[g_i(\mathbf{d}, \mathbf{X}, \mathbf{P})] - \beta_i \sigma[g_i(\mathbf{d}, \mathbf{X}, \mathbf{P})] \geq 0, \quad i = 1, \dots, k_g, \quad (9)$$

$$\sigma[c_k(\mathbf{d}, \mathbf{X}, \mathbf{P})] \leq \sigma_k'', \quad k = 1, \dots, k_c, \quad (10)$$

$$d_j^l \leq d_j \leq d_j'', \quad j = 1, \dots, n_d, \quad (11)$$

$$\mu_{x_r}^l \leq \mu_{x_r} \leq \mu_{x_r}^u, \quad r = 1, \dots, n_x. \quad (12)$$

The weight coefficient $\alpha \in [0, 1]$, found in formula (8), specifies the relevance of each of the criteria, whereas μ^* and σ^* are normalizing constants. Assuming $\alpha = 0$, the optimisation problem is converted into a typical task of minimising the mean value. For $\alpha = 1$, however,

Figure 2. The concept of constraints in the robust optimisation.

optimisation becomes the problem of minimising the objective function variance. A key element of the algorithm employed to perform the robust optimisation task involves an effective method of estimating mean values and standard deviations of the objective function and the constraint function [Stocki et al. (2012)].

3 EXAMPLE

In the study, a steel single-layer lattice dome loaded with force $P = 3.665$ kN at each node was analysed. The members of the dome were designed to be made of S235 steel, with yield strength $f_y = 235$ MPa, Young's modulus $E = 210$ GPa and Poisson index $\nu = 0.3$. The geometry of the dome of concern is shown in Figure 3 and Table 1. The weight of the structure modelled is $Mass = 813.5139$ kg.

In the structure, three groups of bars were specified as shown in Table 2.

Figure 3. The lattice dome geometry.

Table 1. The lattice dome geometry.

Node number	X	Y	Z	Node number	X	Y	Z
	[cm]	[cm]	[cm]		[cm]	[cm]	[cm]
1	646.716	646.716	46.000	10	378.837	1293.431	0.0
2	646.716	996.716	34.000	11	914.594	1293.431	0.0
3	894.203	894.203	34.000	12	1293.431	914.594	0.0
4	996.716	646.716	34.000	13	1293.431	378.837	0.0
5	894.203	399.228	34.000	14	914.594	0.0	0.0
6	646.716	296.716	34.000	15	378.837	0.0	0.0
7	399.228	399.228	34.000	16	0.0	378.837	0.0
8	296.716	646.716	34.000	17	0.0	914.594	0.0
9	399.228	894.203	34.000				

Table 2. Cross-sections adopted for individual groups of bars.

Group name	Bar numbers	Profile
Meridian_1 (Blue)	1 to 8	RO54 x3.6
Meridian_2 (Orange)	17 to 32	RO60.3x6.3
Ring (Green)	9 to 16	RO51x6.3

Table 3. Values of axial forces and load capacity for the most stressed members of the structure and values of the maximum vertical and horizontal displacements for nodes 2-9.

Internal force/Load capacity	Meridian_1 Bar no. 7	Meridian_2 Bar no. 30	Ring Bar no. 15
Ned [kN] – axial force	12.664	25.093	31.976
Strength utilisation [%]	46	55	56
Maximum vertical displacement [mm]	1.071		
Allowable vertical displacement [mm] – D/300	4.667		
Maximum horizontal displacement [mm]	0.06		
Allowable horizontal displacement [mm] – H/150	0.307		

Table 3 collates the values of internal forces and load capacity for the most stressed elements of individual bar groups and limit values of displacements of nodes 2 to 9.

3.1 Reliability analysis

The first stage involved structure reliability analysis in which the Monte Carlo method was used. Geometric characteristics of cross-sections (A_i - area of successive bar groups) and heights of individual structure nodes (Z_i) were assumed as random variables. The description of random variables is shown in Table 4. The variables are not correlated.

The structure under analysis is susceptible to stability failure resulting from the condition of the node snap-through. On this basis, a displacement limit function was adopted, which refers to the maximum displacement value at the instant of the node snap-through:

$$f_s = 1 - \frac{w(\mathbf{x})}{w_{\max}} = 1 - \frac{w(\mathbf{x})}{1.841} \tag{13}$$

where: $w(\mathbf{x})$ – node 2 displacement, $w_{\max}$ – node 2 displacement at the instant of the snap-through.

The value of the reliability index, at the instant the structure was loaded with force $P = 3.665$ kN, was $\beta = 2.138$, whereas the failure probability was $p_f = 0.016$.

3.2 Deterministic optimisation

The second stage involved the deterministic optimisation. The design variables were cross-sectional areas of the individual bar groups. The following notation was adopted for the variables: Meridian_1: $A1$, Meridian_2: $A2$, and Ring: $A3$. The objective function was the structure weight:

Table 4. Description of random variables.

Random variables X_i	Mean values	Standard deviation	Variation coefficient	Random variables X_i	Mean values	Standard deviation	Variation coefficient
	[cm]	[cm]	[%]		[cm]	[cm]	[%]
A_1	5.7	0.285	5	$Z4$	34	0.34	1
A_2	10.7	0.535	5	$Z5$	34	0.34	1
A_3	8.85	0.443	5	$Z6$	34	0.34	1
$Z1$	46	0.46	1	$Z7$	34	0.34	1
$Z2$	34	0.34	1	$Z8$	34	0.34	1
$Z3$	34	0.34	1	$Z9$	34	0.34	1

$$f_C = \text{minimum}(Mass) \quad (14)$$

Simple constraints are shown in Table 5. They constitute the upper and lower boundary of the design variables being sought.

The inequality constraint was formulated as a condition of non-exceedance of the allowable node vertical displacement:

$$g(\mathbf{x}) = w - w_{\max} = w - 1.841 < 0 \quad (15)$$

The deterministic optimisation was carried out using the Nelder Mead simplex method at the maximum number of iterations $N = 1000$, and the convergence parameter $\epsilon = 1.0E-08$.

The dimensions of cross-sections obtained for individual bar groups are summarised in Table 6. The value of the objective function was $f_C = 772.924$ kg. The failure probability and the reliability index, which in this case amounted to: $p_f = 0.034$, $\beta = 1.824$, respectively, were also verified.

3.3 Robust optimisation

As regards robust optimisation, random variables, design variables, the objective function and constraints were defined. The value of the variation coefficient was set at 5%. For the case of concern, the robust optimisation task takes on the following form:

1. Find the values of variables: $A1, A2, A3$
2. That minimise: $f_C = \frac{1-\alpha}{\eta^*} E[Mass] + \frac{\alpha}{\sigma^*} \sigma[Mass]$
3. With the constraints:

$$\begin{aligned} E[w-1.841] - \beta \cdot \sigma[w-1.841] &\geq 0 \\ 5.415 &\leq R1 \leq 5.985 \\ 10.165 &\leq R2 \leq 11.235 \\ 8.408 &\leq R3 \leq 9.293 \end{aligned}$$

where: $\alpha \in [0, 1]$ – specifies the relevance of each of the criteria, η^*, σ^* - normalizing constants, $w-1.841$ – constraint on the allowable vertical displacement.

The dual response area method and the strategy of statistical approximation were employed. The parameters adopted were: $\alpha = 0.5$, $\beta = 2.0$. Dual response areas were

Table 5. Description of simple constraints.

Design variable	Lower boundary [cm]	Upper boundary [cm]
$A1$	5.415	5.985
$A2$	10.165	11.235
$A3$	8.408	9.293

Table 6. Values of the design variables produced with deterministic optimization.

Random variable	Optimum value [cm]
$A1$	5.415
$A2$	10.167
$A3$	8.408

Table 7. Values of design variables obtained in the robust optimisation.

Random design variable	Optimum mean value [cm]
$A1$	5.426
$A2$	10.216
$A3$	8.793

constructed using the the Kriging method, while the experiments were designed in accordance with optimum Latin hyper-cube sampling. To solve the optimisation problem of concern, i.e. to minimise the f_C function, the Nelder Mead simplex algorithm was applied.

The robust optimisation yielded the following values of the design variables shown in Table 7. In this case, the structure weight was $f_C = 782.076$ kg. The robust optimisation type produced the structure weight that was slightly higher compared with that obtained with the deterministic optimisation. The reliability index increased and the failure probability decreased compared with the deterministic optimisation, for which the values were $\beta = 1.951$ and $p_f = 0.026$.

4 SUMMARY

In the example above, two methods of structure optimisation, based on analogous constraints and objective function, were compared. At the initial stage of the design, the cross-section dimensions adopted were $A1 = 5.7 \text{ cm}^2$, $A2 = 10.7 \text{ cm}^2$, $A3 = 8.85 \text{ cm}^2$. The reliability index for this design situation was $\beta = 2.138$. In the deterministic optimisation carried out with the Nelder-Mead simplex method, new dimensions of cross-sections were determined. A decrease in the structure weight by approx. 5% was observed. Modification of the areas led to a decrease in the reliability index to the value of $\beta = 1.824$.

As a result of robust optimization, a structure with a slightly larger mass was obtained. The difference in the weight of the structure in the case of deterministic and immunity optimization did not exceed 2%. However, the reliability index, which measures the safety of the structure, has increased significantly. At the expense of a slight increase in weight, we obtained a structure that is more reliable and resistant to the dispersion of parameters characterizing the structure's operation.

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