Selected aspects of designing the cold-formed steel structures

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ABSTRACT: This paper addresses the selected issues of designing the cold-formed steel structures in the form of shells and bars such as facades, purlins, wall rails and main bearing capacity members. In first part of the study particular consideration is paid on stability of steel structures with a reference to the theoretical background of global, local and distortional buckling. To study the most typical stability response of structures with imperfections using nonlinear algebraic equilibrium equations which strictly describe pre- and post-buckling behaviour of structures a basic bar model is analysed. The second part of the article presents the transition from the von Kármán scientific theory to the first semi-empirical approach known as “Winter formulas” and the formulas adopted in EN 1993-1-3 and EN 1993-1-5, to be currently used in structural engineer practice. Then, the particularities in the verification of the resistance and different modes of buckling of the cold formed profiles with stiffened or unstiffened walls are discussed, through theoretical calculations and test-assisted calculations. Finally, the typical applications of cold formed products are presented.

1 INTRODUCTION

Research on the development of theories and methods of designing thin-walled cold-formed steel structures is currently carried out in many centres and covers a very wide range of problems. The use of cold formed steel sections is very advantageous from an economic point of view, however, the design of this type of structure presents many difficulties due to their high sensitivity to initial geometric imperfections and their tendency to local instability. The commonly used beam theory proposed by Timoshenko (1961) and developed by Vlasov (1963) does not allow taking into account local effects. In turn, it is possible using the shell theory in FEM and the interactive buckling analysis, which may take the form of local buckling, distortional, flexural, torsional or flexural-torsional. Although FEM analysis allows for a comprehensive analysis of the entire structural system, it is still a very time-consuming process. In Chou & Rhodes (1997) reviewed the work on plate buckling with particular emphasis on experimental work on thin-walled plates and beams. Some of the experimental results were selected comprehensively and exhaustively, and then developed to form a database for use by future researchers. The current state of knowledge on the rational design of steel structures made of cold-formed sections, both in relation to calculation models and to construction issues, is presented in Bródka et al. (2006). It includes the latest results of experimental tests of bent sheet structures and discusses a number of complex calculation models for thin-walled structures proposed in European standards. A thorough analysis of the works carried out in the last few years in the field of designing thin-walled steel structures was also carried out by Schafer (2011). He put particular emphasis on the review of currently used calculation

methods together with the analysis of their effectiveness. In addition, he presented the progress in the application of individual analysis and design methods and indicated future directions for the development of cold-formed steel structures, at the forefront of which are issues related to geometric imperfections. In the case of cold formed steel members, the initial geometric imperfections have a significant influence on the stability response of the structure. There are many works that present the different types of geometric imperfections characteristic of cold-formed structures and the ways of introducing them into the numerical model. One of the most common methods of accounting for initial geometric imperfections, dominant in design standards, is the use of an appropriate equivalent load or initial deflection Kozłowski (2008), Murzewski (1996). This approach is based on a second-order analysis in which the effects of residual stresses and initial geometric imperfections in individual elements are taken into account in the global analysis. In Barszcz & Gizejowski (2007) the authors proposed an alternative method of imperfection modelling, based on the concept of equivalent bar stiffness. In this method, the bar is treated as a hypothetically simple element, and its stiffness is determined at each load level. This approach does not require explicit modelling of equivalent geometric imperfections or equivalent forces. The methods presented above work only in the case of global imperfections, while in the case of local imperfections it is necessary to use the shell models. Then it is possible to introduce to the initial configuration of the bar deformed geometry obtained on the basis of measurements of actual imperfections or standard tolerances. However, this method is very laborious, especially when the bar is FEM-modelled with shell elements. Moreover, due to the large randomness of imperfections, the obtained results are not always reliable. The problem of the plausibility of the results of numerical analyses concerning the stability of the structure was undertaken in Schafer & Peköz (1998) examining the accuracy of the computational models taking into account the initial geometric imperfections and residual stresses and comparing the obtained results with the results of experimental tests. In the works Dubina & Ungureanu (2001) the influence of local geometric imperfections of thin-walled sections on the stability of columns and short beams was analysed. Particular attention was paid to the characterization and classification of imperfections in order to develop convenient methods of their introduction into nonlinear FEM models. Since the most dangerous imperfections are those whose shape corresponds to the buckling modes, a very effective method of considering them can be the method of generating the shape of the deformed geometry of the initial nonlinear analysis tasks as a linear superposition of the buckling modes obtained from solving the linear eigenvalue problem Rzeszut et al. (2004). In this method, the amplitudes of the buckling modes are scaled with proportionality coefficients determined from actual imperfections measurements using the Galerkin-based error minimization. The studies by Garstecki & Rzeszut (2009) describe the effectiveness of the above method in terms of the number of measurement points and the manner of their distribution along the bar length, minimizing the approximation error. It is worth noting that the amplitude and shape of the imperfections strongly affect the stability of the structure. Therefore, the correct modelling of the imperfection corresponding to the actual measured geometrical imperfections plays a key role in the stability analysis and design of thin-walled members. This problem is related to another dangerous phenomenon, interactive buckling. The interaction of the global ultimate load capacity and the local critical load capacity was investigated by (Kowal 2001). He pointed out that in the case of an ideal column, the loss of local and global stability may occur separately, which corresponds to the assumption of uncoupled equations. However, in real structures, burdened with imperfections, there is a mutual influence of local and global stability. The author criticized the commonly applicable criterion of the initial column bending as a condition for technical acceptance of the structure and proposed the introduction of a new constraint, which is taking into account the influence of interactive buckling. The interaction of global and local stability loss occurs when the critical loads corresponding to local and global buckling are equal or slightly different. This phenomenon may lead to a reduction in the value of the critical load and to the unstable post-critical behaviour of the structure. Due to the high sensitivity of the structure to initial imperfections, the stability analysis should be carried out with appropriate consideration of the initial geometric imperfections, both local and global.

2 MECHANICS OF THIN-WALLED, COLD-FORMED MEMBERS IN STABILITY PROBLEMS

The main mechanism causing the failure of thin-walled steel structures is the loss of stability. The broadly understood stability of a structure is defined as a phenomenon in which a slight disturbance causes a very large change. Depending on the effect of the disturbance, the condition of the structure can be defined as stable or unstable. Lapunov, who investigated the stability of differential equations using two separate methods, direct and indirect, is considered to be the creator of this definition. The indirect method is based on the assumption that the explicit form of the solution of the differential equation is known. On the other hand, the more general direct method does not require knowledge of an explicit solution. The works of Lapunov were developed by Bogusz (1972), who formulated the concept of technical stability.

Another approach to the analysis of the equilibrium states of the structure is the energy approach, consisting in the analysis of the potential energy, which is the basis for determining the equilibrium state of the structure. The structure is in equilibrium when the potential energy reaches a minimum. The necessary condition for the occurrence of the minimum potential energy is the disappearance of its first variation. This condition can be written as:

$$\delta\Pi = 0 \quad (1)$$

where Π is the potential energy of the system.

The nature of the equilibrium path is determined by the higher order variations of potential energy. The type of bifurcation point can be determined based on the analysis of the second variation of the potential energy according to:

$$\delta^2\Pi \begin{cases} > 0 & \text{stable} \\ = 0 & \text{neutral} \\ < 0 & \text{unstable} \end{cases} . \quad (2)$$

In the classical theory of stability, the type of equilibrium path can be illustrated by a ball moving on a differently shaped surface. Figure 1a shows the stable equilibrium. After any position disturbance, the ball always returns to its initial position, which corresponds to the minimum of potential energy. Unstable state of equilibrium is illustrated in Figure 1b: the initial position of the ball corresponds to the maximum of potential energy, and a small impulse changing the position of the ball causes an imbalance until the minimum potential energy is reached. Figure 1c shows the state of neutral equilibrium in which the ball changes its position by rolling on a horizontal plane without changing the potential energy. In the problems with many degrees of freedom, there are also other cases, e.g. the metastable point (Figure 1d, e). It is a point on a curve or surface characterized by local minimum of potential energy, that in any of its surroundings, the ball may follow the stable or unstable path of equilibrium. This surface is generally referred to as a hypersurface.

In stability studies, the primary task is to identify bifurcation points or limit points, since it is at these points that the structure is lost. The bifurcation point can be determined on the basis of solving the eigenvalue problem, as a result of the critical load value and a normalized

Figure 1. States of equilibrium: a) stable, b) unstable, c) neutral, d), e) metastable.

eigenvector are obtained. Based on the critical load value, only multidirectional equilibrium path is determined, but no information is obtained about the post-critical behaviour of the structure. Therefore, it is extremely important to define the nature of the bifurcation point. For this purpose, an analysis of post buckling should be performed by determining the higher-order variation of potential energy. Hence, a stable and symmetrical bifurcation point can be distinguished when the following conditions are met:

$$\delta^2\Pi = 0, \quad \delta^3\Pi = 0, \quad \delta^4\Pi > 0, \quad (3)$$

and an unstable bifurcation point when:

$$\delta^2\Pi = 0, \quad \delta^3\Pi = 0, \quad \delta^4\Pi < 0. \quad (4)$$

Moreover, when:

$$\delta^2\Pi = 0, \quad \delta^3\Pi > 0, \quad \delta^4\Pi < 0, \quad (5)$$

the bifurcation point is asymmetric. It is worth noting that reaching the bifurcation point should not always be associated with the failure mechanism of structure. Equilibrium paths can take a more complex form when there is more than one minimum of potential energy. Usually it is associated with so called snap-through phenomenon based on transition from the local to the global minimum of potential energy. Commonly it is associated with the release of internal energy due to a sufficiently large impulse and the transition of the structure to a lower energy level. In thin-walled steel structures, this phenomenon is accompanied by a loud sound effect. The impulse to snap-through phenomenon is usually caused by the specific configuration of initial geometric imperfections, difficult to predict and define. Therefore, snap-through analysis presents a challenging engineering issue and creates a lot of design problems (Zhao et. al 2008). In the case of variable loads, a multiple snap-through may cause material fatigue and, consequently, its brittle failure. An excellent illustration of this behaviour can be found in the example of the Mises truss of small height (Figures 2a), in which the bars are elastic, and the deformations are symmetrical. When imposing ever-greater deflections u , the value of the force P can be calculated using the equilibrium equation. Hence P is a function of u . Note that the part of the curve (1-2) represents the unstable equilibrium path. In the case of the gravitational action of force P , the snap-through takes place from point 1 to point 1' located on the stable equilibrium path 1'-4. In the unloading process, the structure follows the equilibrium path from point 4 through point 1' to point 3, for which the force $P = 0$ is related to the displacement $u = 2f$. This is an example of an irreversible process. Full non-linear equilibrium paths for Mises's truss were presented by Kwaśniewski (2009). As a result of experimental studies on slender steel girders Rzeszut et al. (2009) it was shown that the snap through can be a fully reversible process (Figure 2c).

The above considerations concern the global stability of bars. Thin-walled bars, depending on the ratio of their width to thickness, may additionally lose their local stability under the influence of compressive or shear stresses. When considering the general stability, the Vlasov theory is used, in which a model of a bar with an open cross-section is assumed, which meets the conditions of the rigid contour hypothesis taking into account warping. When considering local stability, it is assumed that the member axis remains undeformed, while the contour is distorted. Thus, the rigid contour hypothesis is rejected here. In addition to the local loss of stability, distortion instability is observed. The analysis of the local loss of stability is carried out on the basis of the theory of plates and shells, the foundations of which were formulated by, inter alia, Timoshenko, Bleich and Brzoska. Numerical analysis of thin-walled bars is carried out using the shell model in Finite Element Method (FEM) or in the Finite Strip Method (MPS). Figure 3 illustrates the phenomena related to the stability of thin-walled elements. It presents the critical stresses of an axially compressed thin-walled bar determined with the use

Figure 2. Snap through phenomenon; a) model of a structure and equilibrium paths for Mises truss Rzeszut (2015), b) results of experimental tests Rzeszut et al. (2009).

of FEM beam and shell elements and the Vlasov theory. In the case of very slender bars, characterized by the global loss of stability, all results obtained by FEM are in great agreement with the Vlasov beam theory.

However, in the case of bars, where first buckling mode is local, the application of Vlasov theory leads to very large errors. Therefore, in the case of thin-walled steel members, an individual numerical analysis based on the shell models is necessary. However, despite the ever-faster developing numerical technologies and despite the increase in computing power of computers, the analysis of the entire real structure using the shell models is still too time-consuming and impractical. In engineering applications, torsion and plate stability are analysed separately. This approach can be dangerous when assessing the bearing capacity of real thin-walled, cold-formed steel structures. Therefore, when assessing the load bearing capacity of elements according to Vlasov theory, the influence of loss of cross-sectional walls stability should be additionally investigated in accordance with the plate theory. The local loss of

Figure 3. Critical stresses of an axially compressed thin-walled bar determined with the use of FEM beam and shell elements and the Vlasov theory, Rzeszut (2015).

stability is also related to the post-critical load bearing capacity, which is the ability of the cross-sectional walls to continue load bearing capacity in the safe range after reaching a critical state. The condition for such behaviour of the wall is that at least one longitudinal edge is stable. It needs to be highlighted that multimodal post-critical analysis of thin-walled bars allows for the determination of interactive buckling, which may lead to a reduction in the critical load value, to unstable post-critical behaviour and to a high sensitivity to imperfections.

3 NUMERICAL EXAMPELS

3.1 Model structures

In order to analyse the complex cases of initial imperfection interactions, a model structure was developed consisting of four perfectly rigid bars connected by linear elastic hinges with the rotational stiffness k_n where $n=1,2,3$ (Figure 4).

It allows for derivation of close form formulas accounting for large displacements in a form of a set of three nonlinear equations of equilibrium:

$$\begin{cases} k_1 \left[as \frac{u_1}{L} - as \frac{u_2 - u_1}{L} \right] = Pu_1 \\ k_2 \left[as \frac{u_2 - u_1}{L} - as \frac{u_3 - u_2}{L} \right] = Pu_2 \\ k_3 \left[as \frac{u_3 - u_2}{L} - as \frac{u_3}{L} \right] = Pu_3 \end{cases} \quad (6)$$

The above equations represent a non-linear eigenvalue problem in which the displacement of nodes u_n is unknown. The initial geometrical imperfections are introduced into the model in the form of initial displacement v_n^i and do not induce any stress. Thus the total displacements take the form:

$$u_n = u_n^i + u_n^e. \quad (7)$$

The terms with superscript e represent elastic part of displacements and with superscript i initial geometrical imperfections. Numerical examples are solved for various amplitudes of initial imperfections (Eq. 8) introduced as a linear combination of eigenvectors u_m^{cr} obtained from the linear eigenvalue problem scaled by mode participation factor a_m :

$$u_n^i = a_m \cdot u_m^{cr}. \quad (8)$$

From among many special cases studied by the authors, let us consider the example shown in Figure 5.

Figure 4. Geometry of model structure, Rzeszut (2015).

Figure 5. Equilibrium paths obtained for initial geometric imperfections developed as superposition of: low amplitude of the first and medium amplitude of the second or third buckling mode (m1s2 or m1s3); displacement of the node: a) No. 1, b) No. 2, c) No. 3, Rzeszut (2015).

This example shows the equilibrium paths $\lambda - \bar{u}_i$, where $\bar{u}_i$ is normalized displacement of i^{th} node with respect to bar length and λ is buckling load multiplier obtained for small amplitudes of geometric imperfections developed according the shape of the first buckling mode (m1) and medium amplitude of second buckling mode (s2) or third one (s3). The presented paths describe the equilibrium paths in node 1, 2 and 3 respectively. It is worth to notice that in all three cases stable, unstable and metastable equilibrium states are observed. Moreover two snap-through and thus two bifurcation points are observed: the first is related to the first and the second is related to the second or third buckling mode.

The analysis of displacements before and after the snap-through shows that the initial imperfection, being a superposition of two different buckling modes, strongly influences the shape of the displacement vector. In Figure 6 pre- and post- snap-through displacement configuration for case m1s2 is presented. It can be seen that at the first bifurcation point (snap-through from point 1 to point 1') the decisive factor is the first buckling mode, and at the second bifurcation point (snap-through from point 2 to point 2') shape of displacement corresponds to the second buckling mode.

On the other hand in Figure 7 pre- and post- snap-through displacement configuration for case m1s3 is presented. As it was shown in the previous case the first bifurcation point (snap-through from point 1 to point 1') the decisive factor is the first buckling mode, but at the second bifurcation point (snap-through from point 3 to point 3') shape of displacement

Figure 6. Pre- and post- snap-through displacement configuration for case m1s2, Rzeszut (2015). a) snap-through from point 1 to point 1', b) snap-through from point 2 to point 2'.

Figure 7. Pre- and post- snap-through displacement configuration for case m1s3, Rzeszut (2015). a) snap-through from point 1 to point 1', b) snap-through from point 3 to point 3'.

corresponds to the third buckling mode. Thus, the type of initial geometric imperfection strongly influences the nature of the equilibrium path.

4 BASIS OF EFFECTIVE WIDTH OF COMPRESSED PLATE CALCULATION

4.1 Theoretical basis

The following differential equation for buckling of a plate was introduced by von Kàrmàn:

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial^2 x \partial^2 y} + \frac{\partial^4 w}{\partial y^4} = \frac{1}{D} \left(N_x \frac{\partial^2 w}{\partial x^2} + 2 N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} \right) \quad (9)$$

where: $D = \frac{Et^3}{12(1-\nu^2)}$, N_x , N_y and N_{xy} are forces applied uniformly to the edges of the plate. In the case where the plate is subjected to a load N_x uniformly distributed on edge a, the deflected shape is shown in the Figure 8.

The minimum potential energy method, with the series:

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \left(\sin \frac{\pi m x}{a} \sin \frac{\pi n y}{b} \right) \quad (10)$$

gives the following solution:

Figure 8. Rectangular plate subjected to compression stress.

$$N_{cr} = \frac{\pi^2 a^2 D}{m^2} \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^2. \quad (11)$$

When the b dimension is relatively large, the minimum value of the critical force N_x is obtained with $n = 1$ and $b = a$, which leads to the formula:

$$\sigma_{cr} = \frac{N_{cr}}{t} = E \left(C \frac{t}{b} \right)^2 \quad (12)$$

where:

$$C = \frac{\pi}{\sqrt{3(1-\nu^2)}} = 1.90. \quad (13)$$

The distribution of the stresses over the width of the plate is not uniform, because the interior part undergoes a local buckling. The more the pressure applied to the plate increases, the more the stress in the middle of the wall decreases:

Figure 9. Local buckling in a compressed wall.

This observation led von Kàrmàn in 1932 to the concept of “effective width”, where:

- the central part of the wall under buckling is neglected,
- the total load is carried by the external parties.

In this approach, it is assumed that on the edges the yield stress f_y is reached and that it is constant over the fictitious effective width derived from (12) by replacing σ_{cr} by f_y and b by b_e :

$$b_e = Ct \sqrt{\frac{E}{f_y}}. \quad (14)$$

From equations (12) and (14) the following relation can be obtained:

$$\rho = \frac{b_e}{b} = \sqrt{\frac{\sigma_{cr}}{f_y}}. \quad (15)$$

4.2 Particularities to consider in order to adapt von Kàrmàn theory to cold formed sections

Von Kàrmàn theory is based, in particular, on the following explicit or implicit assumptions:

- (1) The section is composed of flat walls with sharp corners,
- (2) The thickness of each wall is constant over whole its width,
- (3) The yield stress is constant over whole section,
- (4) There are no residual stresses in the section,
- (5) The edge stress of all walls of the section is constant and equal to the yield stress,
- (6) There are no geometric imperfections in the section.

In the case of cold formed sections, the above assumptions are not satisfied, namely:

- the corners are rounded and (2) the thickness is reduced in rounded corners (Figure 10);
- (3) the yield stress is higher and (4) the residual stress exists in the rounded corners (Figure 11);
- (5) the edge stress is not constant in the webs (Figure 12);
- (6) in real structures, imperfections always exist.

4.3 Empirical improvement of von Kàrmàn theory

Typical example of testing of trapezoidal sheeting behaviour, in order to investigate on the behaviour of sheeting where the upper flanges work in uniform compression and the webs work under stress gradient, is shown in the Figure 12. A typical example of experimental investigation on trapezoidal sheets, in order to verify their behaviour under a bending moment, where the upper flanges work in uniform compression and the webs work under stress gradient, is illustrated in Figure 13.

Based on large experimental and theoretical investigation, Winter (1947) found that Eq. (16) is applicable when the edge stress of the wall is $\sigma < f_y$:

$$b_e = Ct\sqrt{\frac{E}{\sigma}}. \quad (16)$$

Subsequently, he demonstrated that there is a linear relation between C and $t/b_p\sqrt{E\sigma}$ and consequently he proposed the following value for the term C :

$$C = 1.9 \left(1 - 0.475 \frac{t}{b_f} \sqrt{\frac{\sigma_{cr}}{f_y}} \right) \quad (17)$$

where b_f is explained in the Figure 10.

Figure 10. Rounded corner between two adjacent walls.

Figure 11. Effect of strain hardening in the cold worked corner.

Figure 12. Distribution of stresses in asymmetric sheets.

Figure 13. Example of testing of trapezoidal sheeting under bending moment.

Therefore, the equation (16) takes the form:

$$b_e = 1.9t\sqrt{\frac{E}{\sigma}}\left(1 - 0.475\frac{t}{b_f}\sqrt{\frac{E}{\sigma}}\right) \quad (18)$$

so, the equation (16) may be written as a function of ratio $\sigma_{cr}/\sigma = \rho$ as follows:

$$\rho = \sqrt{\frac{\sigma_{cr}}{\sigma}} \left(1 - 0.25 \sqrt{\frac{\sigma_{cr}}{\sigma}} \right). \quad (19)$$

After a long-accumulated experience, the equation (19) was refined as follows:

$$\rho = \sqrt{\frac{\sigma_{cr}}{\sigma}} \left(1 - 0.22 \sqrt{\frac{\sigma_{cr}}{\sigma}} \right). \quad (20)$$

The equation (18) can also be presented as:

$$b_e = \rho b_f \quad (21)$$

where:

$$\rho = \frac{1 - 0.22/\lambda}{\lambda}, \quad (22)$$

$$\lambda = \frac{1.052 b_p}{t} \sqrt{\frac{\sigma}{E k}}. \quad (23)$$

The formulae (21) to (23) are used in AISI Specification and in North American Specification. A similar formulation, although more universal, is adopted in EN 1993-1-5:

$$\bar{\lambda}_p = \sqrt{\frac{f_{yb}}{\sigma_{cr}}} \frac{b_p}{t} \sqrt{\frac{12(1 - \nu^2) f_{yb}}{\pi^2 E k_\sigma}} \cong 1,052 \frac{b_p}{t} \sqrt{\frac{f_{yb}}{E k_\sigma}} \cong \frac{b_p/t}{28,4 \varepsilon \sqrt{k_\sigma}} \quad (24)$$

where: b_p is explained in the Figure 10, k_σ is the buckling factor corresponding to the stress ratio $\psi = \sigma_2/\sigma_1$, σ_1 and σ_2 are the edge stress of the wall, $\sigma_2 \leq \sigma_1$, $\varepsilon = \sqrt{\frac{235}{f_y}}$.

When $\sigma_{com,Ed} < f_{yb}/\gamma_{M0}$ the coefficient ρ is defined as follows:

$$\bar{\lambda}_{p,red} = \bar{\lambda}_p \sqrt{\frac{\sigma_{com,Ed}}{f_{yb}/\gamma_{M1}}} \quad (25)$$

$$\rho = \frac{1 - 0,055(3 + \psi)/\bar{\lambda}_{p,red}}{\bar{\lambda}_{p,red}} + 0,18 \frac{(\bar{\lambda}_p - \bar{\lambda}_{p,red})}{(\bar{\lambda}_p - 0,6)} \text{ but } \rho \leq 1 \quad (26)$$

4.4 Taking into account of distortional buckling

4.4.1 Distortional buckling of plane walls with stiffeners

The examples of distortional buckling and the calculation model for elements with intermediate and edge stiffeners are shown in the Figure 14.

According to this model the elastic critical buckling stress in the stiffener is defined as for an infinite long beam on elastic foundation under constant compression:

$$\sigma_{cr,s} = \frac{2\sqrt{KEI_s}}{A_s} \quad (27)$$

where: A_s and I_s are respectively area and second moment of area of the stiffener. The buckling coefficient is determined in function of the relative slenderness $\bar{\lambda}_d$

$$\bar{\lambda}_d = \sqrt{f_{yb}/\sigma_{cr,s}}. \quad (28)$$

4.4.2 Distortional buckling of section

The examples of distortional buckling of sections are shown in the Figure 15.

A particular case of distortional buckling and the calculation model for beams under bending, with one flange held as shown in the Figure 16, was studied by Sokol (1979, 1988, 1995, 2000) and the results obtained have been incorporated into EN 1993-1-3.

Unlike the case of buckling of the stiffener, in this case is taken into account the variation of the compression in the member, depending on the bending moment distribution, as shown in the Figure 17.

The solution is found by expressing the total potential energy:

$$E = \int_0^L \left\{ \frac{EI(y'')^2}{2} + \frac{\beta y^2}{2} + \frac{\delta_1 q_1}{2} \left[(\eta - 1) \frac{x^2}{2L} - \eta x + \frac{L}{2}(1 + \eta) \right] (y')^2 + \frac{1}{2} \delta_p P (y')^2 \right\} dx \quad (29)$$

in form of functional $J[u] = F(\xi, u, u', u'') d\xi$ that reaches the minimum when its first variation is zero, expressed by the so-called Euler-Poisson relation:

$$F_y - \frac{d}{dx} F_{y'} + \frac{d^2}{dx^2} F_{y''} = 0 \quad (30)$$

resulting in the following differential equation of equilibrium:

$$\alpha \left(\frac{d^4 y}{d\xi^4} + \pi^4 K y \right) - \pi^2 \delta_1 [(\eta - 1)\xi^2 - 2\eta\xi + \eta + \delta_p \delta_1 \psi + 1] \frac{d^2 y}{d\xi^2} - 2\pi^2 \delta_1 [(\eta - 1)\xi - \eta] \frac{dy}{d\xi} = 0. \quad (31)$$

Figure 14. Examples of distortional buckling modes and determination of spring stiffness for edge and intermediate stiffeners of the plate wall. a) Actual system, b) Equivalent system.

Figure 15. Examples of distortional buckling shapes.

a)

b)

Figure 16. Calculation model for the distortion buckling of the beam with one flange fully supported laterally and partially maintained in rotation. a) Actual system, b) Equivalent system.

a)

b)

Figure 17. Variation of compression in the free flange depending on bending moment. a) Under down load, b) Under up-lift load.

Using the direct Galerkin method, with the functional basis $\Phi_i = \sin \pi i \zeta$, the following system of n equations is obtained:

$$\left[\alpha + \delta_1 \frac{2\pi^2(2 + \eta + 3\delta_1\delta_p\psi)j^2 - 3(1 - \eta)}{6\pi^2(j^4 + K)} \right] a_j + \frac{4\delta_1}{\pi^2(j^4 + K)} \sum_{i=1, i \neq j}^n \frac{ij(i^2 + j^2)}{(i^2 - j^2)} \left[\eta - (-1)^{i+j} \right] a_i = 0 \quad (32)$$

where: $i=1, n; j=1, n$.

The maximum eigenvalue α_{cr} of the matrix corresponding to the equation system (32) defines the critical buckling length L_f :

$$L_f = L\sqrt{\alpha_{cr}}. \quad (33)$$

Finally, several numerical calculations with a series of different parameters led to the following empirical formula:

$$L_f = \eta_1 L_a (1 + \eta_2 R^{\eta_3})^{\eta_4}. \quad (34)$$

where: L_a is the distance between anti-sag bars, or if none are present, the span L of the beam η_1 to η_4 are the coefficients depending on the number of anti-sag bars, the situation of the

span (simple, end or intermediate) and direction of the load (down or uplift), given in Table 10.2 of EN 1993-1-3.

The buckling coefficient χ is defined using the buckling length L_f from formula (34).

5 CONCLUDING REMARKS

In the paper selected aspects of designing of the cold-rolled steel structures were discussed. In the first part of the study special attention was paid on pre- and post-buckling behaviour of structures with various combinations of geometrical imperfections developed in accordance with three consecutive buckling nodes. It was found that small amplitudes of geometrical imperfections developed in form of first, second or third buckling mode are accompanied by bifurcation points corresponding to the stable, unstable or metastable equilibrium paths. Moreover at the bifurcation point it is possible snap through related to the transition of the structure from a higher to a lower level of potential energy. Introduction of the imperfections, developed as a linear superposition of two buckling modes, results in the appearance of two bifurcation points corresponding to the solution obtained for the perfect structure.

In the second part of the paper, first the transition from the von Kármán scientific theory to the first semi-empirical approach known as “Winter formulas” and ending with the formulas adopted in EN 1993-1-3 and EN 1993-1-5 were presented. Then, the particularities to be considered in the verification of the resistance and different modes of buckling of the cold formed profiles with stiffened or unstiffened walls are discussed, through theoretical calculations and test-assisted calculations.

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