Exploring correlations between the compressive modulus and the flexural modulus of asphalt concrete mixtures

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ABSTRACT: Structural performance of the asphalt pavement can be evaluated through the compressive and flexural moduli. Various predictive moduli models have different independent variables as inputs. In this study, two distinct methodologies were used to calculate the compressive modulus (E^*) based on the flexural modulus, temperature, and frequency. The first model involved an analytical approach based on matrix analysis while the second was based on ridge regression. The effects of binder type can be characterized via a component's transformation matrix. The results of statistical modeling showed a linear relationship between E^* and the independent variables. Comparing outputs of statistical modeling and the matrix analysis showed a good relationship with the experimental results of E^* . Such methodologies are proposed to improve the current models.

Keywords: Dynamic modulus, Flexural modulus, Fatigue, Rutting

1 INTRODUCTION

The structural design of asphalt pavement has been changing due to the development of mechanistic approaches. The empirical methods based on the California Bearing Ratio (CBR) were replaced with advanced analytical methods through the application of finite element and layer elastic theories, implemented in empirical-mechanical softwares. Furthermore, new criteria and structural parameters are being developed to more realistically characterize the structural performance of asphalt mix, in comparison to those which are offered by previous methods. One of the structural parameters is the compressive modulus (E^*) of asphalt concrete over a range of temperature and speed combinations, which is often referred to as the dynamic modulus. Witczak proposed a model to estimate E^* based on the volumetric properties of asphalt mix and the rheological properties of the binder. Later, researchers modified the model to include different engineering properties. For example, Witczak 1-37A (1999) produced a model of E^* based on eight input parameters that characterize aggregate particles, asphalt binder rheological properties, binder-aggregate interactions, and loading conditions, as shown in Equation (1).

$$\begin{aligned} \text{Log}E^* = & -1.249937 + 0.029232P_{200} - 0.001767(P_{200})^2 - 0.002841P_4 - 0.058097V_a \\ & - 0.802208 \frac{V_{A_{ef}}}{(V_{A_{ef}} + V_a)} + \frac{3.871977 - 0.0021P_4 + 0.003958(P_{38})^2 + 0.005470P_{34}}{1 + e^{(-0.603313 - 0.313551\log f - 0.393532\log \mu)}} \end{aligned} \quad (1)$$

where

E^* : Compressive modulus of asphalt (in 10^5 psi)

P_{200} : Percentage of passing the No.200 sieve, by total aggregate weight
 P_4 : Percentage retains on the No.4 sieve, by total aggregate weight (cumulative)
 V_a : Percentage of air voids in the mix, by volume
 V_{Aef} : Percentage of effective asphalt binder content, by volume
 P_{38} : Percentage retains on the 3/8 inch sieve, by total aggregate weight (cumulative)
 P_{34} : Percentage retains on the 3/4 inch sieve, by total aggregate weight (cumulative)
 F : Frequency of loading (Hz)
 μ : Asphalt binder's viscosity

To consider the effect of the frequency of loading on the binder, the shear dynamic modulus and the phase angle of the asphalt binder were included, and this is referred to as the Witczak 1-40D model (Equation 2). With the adapted model, the dynamic response of the binder and binder-aggregate interaction is better explained. In other words, the researchers tried to propose integrated models that encompassed volumetric properties of the mixture and rheological properties of the binder in the temperature range at which the binder behaved as a viscoelastic. The use of the viscoelastic properties of the binder was a breakthrough in the development of accurate predictive models because sensitivity analyses carried out by Yousefdoost et al. (2013) showed that intrinsic characteristics related to asphalt binders play a key role in the accuracy of the predicted compressive modulus.

$$\begin{aligned}
 \text{Log}E^* = & -0.349 + 0.754G_A^{*-0.0052} \\
 & \times \left(6.65 - 0.032P_{200} + 0.0027P_{200}^2 + 0.011P_4 - 0.0001P_4^2 + 0.006P_{38} \right. \\
 & \left. - 0.00014P_{38}^2 - 0.08V_a - 1.06 \frac{V_{Aef}}{(V_{Aef} + V_a)} \right) \\
 & + \frac{2.56 + 0.03V_a + 0.71 \frac{V_{Aef}}{(V_{Aef} + V_a)} + 0.012P_{38} - 0.0001P_{38}^2 - 0.01P_{34}}{1 + e^{(-0.7814 - 0.578585 \log G_A^* - 0.8834 \log \delta_A)}}
 \end{aligned} \quad (2)$$

where

E^* : Compressive modulus of asphalt (psi)
 P_{200} : Percentage of passing the No.200 sieve, by total aggregate weight
 P_4 : Percentage retains on the No. 4 sieve, by total aggregate weight (cumulative)
 V_a : Percentage of air voids in the mix, by volume
 V_{Aef} : Percentage of effective asphalt binder content, by volume
 P_{38} : Percentage retains on the 3/8 inch sieve, by total aggregate weight (cumulative)
 P_{34} : Percentage retains on the 3/4 inch sieve, by total aggregate weight (cumulative)
 G_A : Dynamic shear modulus of the asphalt binder (psi)
 δ_A : Asphalt binder's phase angle (°)

Also, Christensen et al. (2003) proposed the Hirsch model (Equation 3). This was based on the volumetric properties of the mix and the shear modulus of the binder. In addition to the complex shear modulus, Al-Khateeb et al. (2006) added a complex shear modulus at the glass point of the binder as a rheological characteristic of the binder (Equation 4). All these predictive models are useful in the analysis and design of asphalt pavements. However, the models cannot be used for all the utility conditions. For example, Sing et al. found that the Hirsch and Al-Khateeb predictive models are reliable at low temperature ranges, while the Witczak models yield accurate outputs at high temperatures. Also, the models show bias at low temperatures and in high air voids. Thus, further investigation is required to improve the accuracy of the predictive models.

$$E^* = P_C \times \left[4,200,000 \left(1 - \frac{VMA}{100} \right) + 3G_A^* \frac{VFA \times VMA}{10,000} \right] + \frac{(1 - K)}{\left[\frac{1 - \frac{VMA}{100}}{4,200,000} + \frac{VMA}{3VFA G_A^*} \right]} \quad (3)$$

$$K = \frac{\left(20 + \frac{VFA \times 3G_A^*}{VMA}\right)^{0.58}}{650 + \left(\frac{VFA \times 3G_A^*}{VFA}\right)^{0.58}}$$

where

E^* : Compressive modulus of asphalt (psi)

VMA: Percentage of voids in mineral aggregates in compacted mix, by volume

VFA: Percentage of voids filled with asphalt binder in compacted mix, by volume

G_A^* : Dynamic shear modulus of the asphalt binder (psi)

$$E^* = 3 \times \left(\frac{100 - VMA}{100}\right) + \left(\frac{\left(90 + 1.45 \frac{G_A^*}{VMA}\right)^{0.66}}{1100 + \left(0.13 \frac{G_A^*}{VMA}\right)^{0.66}}\right) \times G_g^* \quad (4)$$

where

E^* : Compressive modulus of asphalt (psi)

G_g^* : Complex shear modulus of asphalt binder in glassy state in Pa, which is assumed to be GPa

In addition, the other researchers tried to correlate E^* and the other engineering properties of mixtures. As an example, Apeyagi (2001) found that E^* and gradation could be considered as potential rutting specification indicators for quality control and quality insurance purposes in the field. Wang et al. (2011) characterized the effects of binder migration on E^* . The results indicated that E^* decreased as binder migrated. The reason is that the migration changed the binder content in the mix and therefore the viscoelasticity changed. In the top part of the mix, E^* was somewhat higher than in the lower part. In other words, the elastic property of the aggregate was dominant, while the viscous property of the binder controlled the structural performance of the mix in the bottom. As the temperature increased, more binder migrated, which resulted in segregation of the aggregate particles on the upper surface. In parallel with this problem, the structural performance of mix under the bottom transitioned from interlocking aggregate particles into hydrostatic pressure of the binder. As a consequence, E^* drastically reduced as temperature increased or the loading frequency decreased in the samples with the asphalt migration phenomenon.

Although E^* is a key parameter for structural design of asphalt pavement using Mechanistic Pavement Design Guide (MEPDG), more structural parameters exist. Among the main criteria is the flexural modulus (F_M) of the mixture that governs its fatigue characteristics. Similar to E^* , F_M depends on binder type, additive type, content, temperature, frequency of loading, and aggregate gradation (White 2020, Bekele et al. 2019, Moreno et al. 2013, Fu & Harvey, 2007). Therefore, an estimation of the correlation between E^* and F_M can be useful in the development of future performance-related predictive models. Such models can be used for the structural analysis and design of pavements. In addition, the models can be used to calculate E^* based on laboratory or field results of the F_M . In this research, the E^* and F_M of asphalt mixes prepared using different binder types were evaluated. Then, the correlation between compressive and flexural moduli of the asphalt mix was estimated based on the laboratory results using a proposed statistical and analytical model.

2 MATERIALS AND METHODS

Three asphalt binders were used in this study. Table 1 shows the properties of the binders. C320 is a common unmodified bitumen for asphalt production (AS 2008-2013) while M1000 is an acid-modified multigrade bitumen (White 2016). Jetbind™ is a heavily modified proprietary binder for heavy duty pavement surfaces (White and Embleton 2015). Figure 1 shows

the aggregate gradation of mixture, which was typical dense graded asphalt for road and air-field pavement surfacing.

The samples were tested by using a Universal Testing Machine (UTM) and conditioned in its environmental chamber. The conditioning temperatures were 10 °C, 20 °C, and 30 °C. The frequency of loading for cylinder and beam samples for the compressive and fatigue tests were 0.10 Hz, 1 Hz, 10 Hz, and 20 Hz.

Table 1. Engineering properties of the asphalt binders.

Property	C320	M1000	JetBind™
Softening point (°C)	50	60	69
Viscosity at 135°C (Pa.s)	0.55	1.03	—
Elastic Recovery at 0.1 kPa	3.8	42.4	64.40
Elastic Recovery at 3.2 kPa	1	23.70	52.70
Cumulative creep at 0.10 kPa	3.87	0.59	0.19
Creep compliance at 3.20 kPa	4.15	0.87	0.34

Figure 1. Aggregate size distribution of asphalt mixture.

The compressive modulus of the specific asphalt mix was determined in accordance with the American Association of State Highway and Transportation Officials (AASHTO) T 342-11. To determine the flexural modulus, a four-point beam fatigue test was implemented in accordance with AGPT-T233-06. Three replicate specimens were prepared for each test and mean value at each temperature and frequency was chosen as final result.

3 RESULTS AND DISCUSSION

Figure 2 shows E^* and F_M at various temperatures. It can be seen that curve trends were comparable. Due to the use of various types of binder, the E^* and F_M of the samples were different. For example, the F_M of the sample containing C320 which was tested at 10 °C and 0.10 Hz was 6.40 (Figure 2d), while corresponding values of samples prepared with M1000 and Jetbind™ were 7.44 GPa and 7.73 GPa, respectively.

(a) C320

(d) C320

(b) M1000

(e) M1000

(c) Jetbind™

(f) Jetbind™

Figure 2. E^* and F_M of various asphalt mixtures.

In addition, the E^* of the C320 mixture was 9.07 GPa at 10 °C, which decreased by 61% at 20 °C. The reductions were 52% and 42% for the M1000 and Jetbind™ samples. The amount of reduction in F_M was higher than E^* for all the binders. The potential reason is that the construction materials were generally more resistant to compressive strain/stress compared to flexural strain/stress. Since the compressive modulus test is a type of compressive test, the values of E^* were higher than the F_M of samples for all the binders and temperatures.

The reason behind the similar trend of E^* and F_M at various temperatures can be attributed to the Poisson effect of materials in which the compressive and shear stress/strain in one direction can result in strain/stress in the other. Therefore, any change in E^* may influence the other performance-related properties such as F_M . Estimating such correlations can be useful in finding multi-variable predictive models based on performance-related properties, rheological characteristics of the binder, and aggregate size.

It is easy to calculate the F_M based on the E^* via transformational matrices built using the temperature and frequency of testing. In such an analysis, the matrix of E^* is assumed as the input matrix (Figure 3) and the output matrix is F_M . Since the temperature and frequency are identical for both tests, the rows and columns of the matrix are arranged by the results of E^* and F_M .

The transformational matrix converts the F_M value to equivalent E^* value. Since the values of F_M are lower than those of E^* , the component of the transition matrix is greater than 1. Clearly, the components of the transition matrix change as the binder type changes. Thus, comparing the transition matrices of different binders shows the effects of rheological characteristics of the asphalt mixtures.

$$\begin{matrix} & \xleftrightarrow{n} & \\ \begin{matrix} \updownarrow m \end{matrix} & \begin{bmatrix} F_{Mij} & \cdots & F_{Min} \\ \vdots & \ddots & \vdots \\ F_{Mmu} & \cdots & F_{Mnu} \end{bmatrix} & \cdot (\Psi) = \begin{bmatrix} E_{ij} & \cdots & E_{in} \\ \vdots & \ddots & \vdots \\ E_{mu} & \cdots & E_{nu} \end{bmatrix} & \begin{matrix} \updownarrow m \end{matrix} \\ & \xleftrightarrow{n} & \end{matrix}$$

Figure 3. Matrix of E^* and F_M for the mixture.

where n is number of rows, m is number of columns, E_{ij} , E_{nu} , E_{mu} , and E_{in} are components of the matrix of compressive modulus, and F_{Mii} , F_{Mnu} , F_{Mmu} , and F_{Mju} are components of the flexural modulus matrix.

In other words, E^* transforms to F_M via a function of Ψ . Mathematically speaking, Ψ is a transition for two different moduli from the compressive modulus to the flexural modulus. Therefore, Ψ can be a constant value, linear operator, or multi-variable function, depending on the number and trend of parameters included in the matrices. In this study, the input matrix contained F_M at various temperatures and frequencies and the output matrix showed E^* at the same temperature and frequency. In other words, there was one output per input.

The main advantage of the proposed matrix analysis is that the transformational matrix shows the trend of each component in terms of the independent variables, such as the frequency and test temperature. In addition, there is no limitation to the number of components in the input and output matrix. However, it is recommended that a square matrix be built to find the transformational matrix based on a linear operator. The large matrices require mathematical software to find the operator. Figure 4 shows the input and output matrices of the mixture produced, using C320 binder in a square matrix (4×4). For example, the operator E^* of the mix (9.07 GPa) at 10°C and 0.10 Hz transform to F_M of 6.39 GPa at the same temperature and frequency via the operator. Note that this methodology is more efficient for minimum 3×3 matrix (or 9 components in the matrix).

	0.10Hz	1Hz	10Hz	20Hz		0.10Hz	1Hz	10Hz	20Hz
10°C	6.39	9.37	12.65	13.65	·{ψ}=	9.07	14.34	20.48	22.36
20°C	2.67	4.54	7.10	7.99		3.51	6.17	10.37	11.93
25°C	1.86	3.03	5.25	5.57		2.31	3.93	7.45	8.27
30°C	1.23	2.04	3.64	3.98		1.59	2.64	4.89	5.41
Input (F _M)						Output (E*)			

Figure 4. Input and output of C320 mixture.

Figure 5 shows the transformational matrices of the C320, M1000, and Jetbind™ mixtures. The operator transformed values of one engineering property to another. It should be noted that the component of each matrix was different in that it indicated the effect of the binder type. Further, the components varied, which showed the trend of changes at a temperature and frequency sweep. It is obvious that more components in the matrix provide more data for characterization of the trend of components of the transformational matrix. By analyzing the coefficients of the transformational matrices, it is found that E* was between 10% and 160% higher than F_M. On average E* was 41%, 30% and 50% higher than F_M, for the C320, M1000 and Jetbind™ mixtures, respectively.

{ψ}=	1.41	1.53	1.62	1.63
	1.31	1.35	1.48	1.49
	1.24	1.29	1.41	1.48
	1.29	1.29	1.34	1.40
(a) C320				
{ψ}=	1.49	1.44	1.17	1.21
	1.56	1.26	1.10	1.13
	1.63	1.25	1.16	1.12
	1.50	1.38	1.25	1.21
(b) M1000				
{ψ}=	1.46	1.58	1.25	1.27
	1.64	1.45	1.50	1.30
	2.10	1.47	1.14	1.14
	2.60	1.56	1.42	1.12
(c) Jetbind™				

Figure 5. Transformation of matrix of mixtures.

Equation 5 shows predictive models proposed using the Ridge Regression method based on the F_M, temperature, and frequency (independent variable) of mixtures. Table 2 shows the results of the ridge regression modeling adopted in this study. It can be seen that there was a nearly linear correlation between the E* and the independent variables. The effects of the binder types can be seen in terms of coefficients of the equation. For example, the coefficient of the flexural modulus in the C320 mixture was 0.99, which was two times greater than the corresponding coefficient in the M1000 and Jetbind™. Further, the constant value of the Jetbind™ mixture was 14.68, which was almost two times as much as the values in the other mixtures. This difference can be attributed to the high E* in the Jetbind™ (Figure 2).

$$E^* = \begin{cases} 0.99F_M - 0.18T + 0.12f_r + 6.67 & R^2 = 0.90 \text{ C320} \\ 0.42F_M - 0.14T + 0.10f_r + 7.75 & R^2 = 0.88 \text{ M1000} \\ 0.58F_M - 0.33T + 0.16f_r + 14.68 & R^2 = 0.88 \text{ Jetbind}^{\text{TM}} \end{cases} \quad (5)$$

E* and M_F are in GPa; T is temperature (°C) and fr is frequency (Hz)

To compare the outputs of the predictive models and matrix analysis, the average of the components was considered as an operator in each mix. Figure 6 shows the laboratory results

Table 2. Summary of Results of the ridge regression modelling.

Independent variables	Mix	Standard error	t value (scaled)	Pr>t	Significance
Intercept	C320	NA	NA	NA	NA
Temperature		1.30	11.97	$<2e^{-16}$	Significant
frequency		1.33	4.85	$1.22 e^{-6}$	Significant
F_M	M1000	1.30	26.41	$<2e^{-16}$	Significant
Intercept		NA	NA	NA	NA
Temperature		1.26	9.76	$<2e^{-16}$	Significant
frequency	Jetbind™	1.32	4.01	$5.88 e^{-6}$	Significant
F_M		1.25	12.32	$<2e^{-16}$	Significant
Intercept		NA	NA	NA	NA
Temperature	Jetbind™	2.90	9.63	$<2e^{-16}$	Significant
frequency		2.74	3.09	0.00	Significant
F_M		2.95	10.25	$<2e^{-16}$	Significant

(a) C320

(b) M1000

(c) Jetbind™

Figure 6. Relationships between E^* calculated by the proposed model (E^*_{mod}), transformational matrix (E^*_{mat}), and experiments.

of E^* in comparison to the outputs of the predictive models and the matrices. It can be seen that there was a good correlation between the two methods and the experimental results. According to data represented in Figure 5, the predictive models show the higher correlation coefficients compared to the transformational matrixes. However, both methods can provide comparable outputs with the experimental results of E^* .

4 CONCLUSION

Compressive and flexural moduli are different key parameters in the analysis of the structural performance of asphalt mixtures. However, their trends are comparable at temperature and frequency sweeps, which can be correlated together. The correlations between the compressive and flexural moduli of mixtures containing different binder types were evaluated. In this regard, two different methods were used; matrix analysis and statistical modeling. For each mixture, a transformational matrix was proposed to convert the flexural modulus to the compressive modulus. The results showed that the components of the matrix as the operator depend on the binder type, test temperature, and frequency. Also, the results of ridge regression showed that there is a linear relationship between the compressive modulus and the flexural modulus. It was found that there is a satisfying relationship between the outputs of the proposed approaches, the predictive model and the matrix analysis, with the experimental results. The proposed methods can be used for mechanistic analysis of the design of asphalt pavements. However, the flexural modulus cannot simply be measured as a substitute for the compressive modulus, due to the complex interactions between the aggregate skeleton and the viscoelastic behavior of the binders. It is recommended that more variables such as aggregate gradation, binder type, rheological properties of the binder, and the other engineering properties of mixtures are considered in the proposed methods.

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