Stress random distributions on railway subgrade surface under train loads considering irregular shapes of ballast

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ABSTRACT: The morphological features of railway ballast, especially the edges and corners, have a great impact on the additional stress on the subgrade surface induced by train loads. In this paper, a novel shape reconstruction method was adopted to generate an arbitrary number of ballast particles. Then, a three-dimensional Discrete element-Finite element method (DEM-FEM) coupling model of the ballasted track-subgrade, in which the ballasted track was simulated by DEM and the subgrade soil was simulated by FEM, was established to analyze the contact stress at the ballast-subgrade interface. In this work, 150 groups of ballasted track-subgrade numerical models composed of irregular particles with randomly reconstructed shapes were applied to investigate the stress distributions on the subgrade surface under various train axle loads. Besides, the average stress of a 10 cm diameter circular area on the subgrade surface directly under the wheel load was calculated. The results indicated that both the peak and average additional stresses induced by train loads followed normal distributions for various numerical models (i.e., various groups of random ballast particle morphology). Compared with field testing data of the additional stress on the subgrade surface under train loads acquired by earth pressure cell, the larger average stresses were obtained in numerical models, because of direct contact between ballast particles and the subgrade along with stress homogenization by the pressure cell.

Keywords: Ballasted track, additional stress on subgrade stress, ballast shape, DEM-FEM coupling model, stress random distribution

1 INTRODUCTION

Fine-grained soil was widely used as fill on the subgrade in Chinese conventional railway, which forms the substructure of direct contact between the ballast and soil

subgrade. With the growing demand of heavy-haul freight transportation, the loading cycles and subgrade dynamic stress increase. It can result in several kinds of subgrade deterioration during railway operation, posing threats to traffic safety (Xiao et al. 2020). Thus, the research on the characterization of subgrade dynamic response under heavy load condition is of theoretical significance for the reinforcement and maintenance of existing railways.

For the dynamic response inside the subgrade, the calculation method based on Boussinesq solution is widely adopted to obtain the attenuation curves of dynamic stress in the vertical direction (Chen, 2014). Hence, once the dynamic stress distribution on the subgrade surface is determined, the dynamic response of subgrade can be acquired. At present, most research works have revealed that the peak dynamic stress on the subgrade surface shows a saddle-shaped distribution in transverse direction. Relevant research work (Mei et al. 2019) has analyzed the random distribution of the subgrade dynamic stress from the aspects of stiffness change and track irregularity. However, in their studies, the ballast track was taken as the continuum, and the dispersion of stress distribution at ballast-soil interface was ignored.

With the appearance of advanced measurements, such as pressure-sensitive paper (Mchenry et al. 2015) granular material pressure cell (Liu et al. 2017), and tactile sensor (Aikawa, 2009), the contact stress inside the ballast layer was measured. These testing results indicated the angular characteristics of ballast leading to stress concentrations at the sleeper-ballast-soil interfaces. In addition to experimental methods, some scholars established the ballasted track-subgrade model through the Discrete element-Finite element method (DEM-FEM) coupling method to investigate the effect of contact dispersion and particle geometry on mechanical behavior of ballasted track. Three dimensional shapes such as spherical cluster (McDowell and Li 2016), super ellipsoid (Zhao and Zhou 2017) or polyhedron (Huang and Tutumluer 2013) were used to model ballast particles. Nevertheless, most existing reconstruction methods lack statistical analysis on the overall morphological characteristics of real ballast particles. Thus, there are gaps between the reconstructed and real ballast in shape description.

In this paper, a shape reconstruction method was adopted based on the probability density distribution of morphological indices. Through the preparatory work of laser scanning and analysis of ballast particle morphology, 150 groups of ballast samples randomly regenerated, which conform with the morphological characteristics of natural ballast. After that, a three-dimensional DEM-FEM coupling model of ballasted track-subgrade was established. The average stress of a 10 cm diameter circular area and peak stress were obtained and forecasted with statistical method. The results indicated that both kinds of stresses obey normal distributions. In comparison with field testing data acquired by earth pressure cell, the larger average stresses were obtained in numerical models.

2 STATISTICAL ANALYSIS AND RECONSTRUCTION OF BALLAST PARTICLES MORPHOLOGY

In this paper, point clouds of surface contours of ballast particle samples were obtained by 3D laser scanning and analyzed to give a quantitative description of the morphological characterization of real ballast particles. Related literature (Noura et al. 2017) has pointed out that when the particle number exceeded 400, the statistical result of the ballast particles' morphological characterization would tend to be stable. Therefore, a 50 kg ballast sample containing a total of 584 ballast particles sieved according to the current Chinese code (TB/T 2140-2008) for the first-class gravel ballast, and the grading curve of the ballast sample is presented in Figure 1.

(a) Some selected ballast particles sample (b) Grading curve of ballast sample

Figure 1. Selection and grading curve of ballast sample.

In general, global and local shape features should be both considered when characterizing particle morphology (Huang and Tutumluer, 2013; Pen, 2013). Global morphological features mainly refer to the basic geometric dimensions and shape, and generally described with long axis (Φ_1), middle axis (Φ_2), short axis (Φ_3) and sphericity index (Sp^3) (Yan et al. 2016). Φ_1 indicates the distance between the two farthest surface points of the particle. Located on the maximum projection plane perpendicular to Φ_1 , S , the longest dimension is called Φ_2 . And Φ_3 is the maximum dimension perpendicular to Φ_1 and Φ_2 . The schematic diagram of these three axes is shown in Figure 2.

Figure 2. Schematic diagrams of Φ_1 , Φ_2 and Φ_3 .

Sphericity index (Sp) is defined based on the above three indices, which describes the degree of approximation between an irregular particle and a sphere, and can be solved by following Eq. (1):

$$Sp^3 = \frac{\Phi_2 \times \Phi_3}{\Phi_1^2} \quad (1)$$

Local morphological feature mainly focuses on the edges and angularities on the surface of particles, and in this paper, it was quantified by the curvature index (CI). It is found that at least 20% of the points are located on the edge or vertex, and the corresponding local curvature is greater than that on the face. Therefore, the sum of the curvature from the first 20% is taken as the curvature index, as can be written as Eq. (2):

$$CI = \sum_{i=1}^{20\%N} \rho_i = \sum_{i=1}^{160} \rho_i \quad (2)$$

where ρ_i is the discrete local curvature and is arranged in descending order; N is the number of points in each point cloud, and was normalized to 800 for each particle in this paper.

Then, the above five indices of 584 particles were statistically analyzed and their probability density distributions were calculated. However, because of the large number of original point clouds and high nonlinearity of the morphology indices, we applied the Proper Orthogonal Decomposition (POD) to find dominant eigenvectors for describing the main morphological features of ballast particles, and then took these eigenvectors and corresponding shape expansion coefficients ($C_0, \dots, C_{34}$) to reconstruct equivalent particles. After that, the mapping relationship between the shape expansion coefficients and indices of morphological feature was obtained by Radial Basis Function (RBF) Neural Network, so that the specific coefficients could be calculated according to the desired probability distribution of the morphological feature indices. In this way, it's possible to model any number of ballast particles that conform to the desired probability density distribution. The flow chart of ballast regeneration was shown in Figure 3. For this part, more details can be found in the previous literature (Xiao et al. 2020).

Figure 3. Flow chart of ballast shape regeneration.

3 BALLASTED TRACK-SUBGRADE MODEL

In this work, the ballasted track was simulated using DEM, whereas the sleeper and the subgrade soil were modeled by FEM. For the sake of calculation efficiency and accuracy, the surface coupling method was applied to calculate the interaction between two domains. If there are no contact between ballasts and the sleeper, the analysis of DEM and FEM is conducted separately. Once the contact occurs, the FEM interface mesh is copied into the DEM domain as facet elements and the displacement boundary conditions are introduced and the contact forces acting on the facet elements are transferred into FEM domain (Song et al. 2019).

The model size was determined in terms of the current railway track and subgrade design specifications in China. Only half of each structure was modelled along the cross-sectional. Type □ sleeper was used in simulation, with dimensions of 1.25 m×0.3 m×0.23 m (length×width× height). The height, upper and lower surface widths of track bed were 0.35 m, 1.5 m and 2.2 m respectively. The longitudinal length of track bed and subgrade was 0.6 m. For the widths of the subgrade, the upper and lower surface were 3.05 m and 4.65 m separately. In the vertical range, the thickness of the surface layer was 0.6 m, and the bottom layer was 0.4 m. In order to reduce the reflection of stress wave under cyclic loading, viscoelastic damping boundaries with the grid elements (normal stiffness was 1.2×10^8 N/m and damping coefficient was 0.6) of 5 cm thickness were set around the model. The specific dimensions are indicated in Figure 4.

Figure 4. A three-dimensional DEM-FEM coupling model of sleeper-ballasted track-subgrade.

In this work, 150 groups of ballasted track-subgrade numerical models were generated. The number of ballast particles N_i in each sample ranged from 7373 to 7590. Based on the probability density distribution of the morphological feature indices gained by 3D laser scanning, corresponding index values were obtained in the range of each morphological index with the uniform distribution random number. Adopting the particle shape reconstruction method established above, polyhedral particles were modelled in the discrete element program YADE.

The modeling of the sleeper and subgrade were conducted by open-source code OOFEM (Patzák 2012) as FEM parts. The concrete sleeper was simulated with the linear elastic constitutive model, whereas the Drucker-Prager constitutive model was used for the subgrade. Some parameters used in the DEM-FEM modeling are presented in Table 1.

Table 1. Structural parameters in the DEM-FEM coupling model of the sleeper-ballasted track-subgrade.

DEM	Density $\rho/(\text{kg}\cdot\text{m}^{-3})$	Normal volume stiffness $k_n/(\text{N}\cdot\text{m}^{-3})$	Tangential volume stiffness $k_s/(\text{N}\cdot\text{m}^{-3})$	Friction coefficient	Damping coefficient	Time step $\Delta t/(\text{step}\cdot\text{s}^{-1})$	
Ballast	2600	1.82×10^{10}	1.6×10^{10}	0.65	0.03	1×10^{-6}	
FEM	Density $\rho/(\text{kg}\cdot\text{m}^{-3})$	Modulus E/Pa	Poisson's ratio μ	Friction angle /°	Damping coefficient c	Cohesion / kPa	Time step $\Delta t/$ ($\text{step}\cdot\text{s}^{-1}$)
Sleeper	2600	3.0×10^{10}	0.2	-	0.03	-	1×10^{-6}
Surface layer of subgrade	1950	1.2×10^8	0.33	30	0.3	30	1×10^{-6}
Bottom layer of subgrade	1900	6.0×10^7	0.35	20	0.3	20	1×10^{-6}

The train load applied on the rail pad, with a length of 0.305 m, as shown in Figure 4 (a). About 5 sleepers bore the single axle load at a ratio of 0.1:0.2:0.4:0.2:0.1. The load magnitude was calculated as 0.4 times the wheel weight. Therefore, the amplitudes under the axle load of 25 t, 27 t and 30 t were 50 kN, 54 kN and 60 kN respectively. For the sake of computational cost, the loading frequency was fixed at 1 Hz and the duration was one cycle of half-sinusoidal wave.

4 PROBABILITY ANALYSIS OF DYNAMIC STRESS ON THE SUBGRADE SURFACE

4.1 Random characteristics of dynamic stress on the subgrade surface

The case of the model under 30 t axle load was selected as an example for the following analysis. The distribution of sleeper stress and force chains of ballast particles at the peak of the train load were shown in Figure 5. As can be seen, the distribution of contact stress between the sleeper and ballast was discrete, and stress concentration at the end of the sleeper was induced. The results are consistent with field measurements (Mchenry et al. 2015).

Figure 5. The distribution of sleeper stress and force chains inside ballasted track under 30 t train load.

As illustrated in Figure 5, main force chains were randomly generated inside the ballasted track below the end of the sleeper, which bore the main load and transferred it to the subgrade surface. Within these regions, the interactions between the irregular ballast particles and the subgrade surface were dominated by face–face contact, while in the other areas were edge–face or vertex–face contact. It can be seen that the random formation of the main force chains and the discrete contact at the interface contributes to the random characteristics of dynamic stress on the subgrade surface.

Figure 6 shows the stress nephogram on the subgrade surface under the 30 t axle load, which reveals that, under the loading area, the dynamic stress on the subgrade surface was significantly higher than that at other positions, and the peak dynamic stress was 2165.2 kPa. Besides, in the field experiment, the earth pressure cell with diameter no less than 10 cm is often used to measure the subgrade stress, so the average stress of a 10 cm diameter area on the subgrade surface directly under the wheel load was calculated (hereinafter referred to as “stress at D10 area”). Under the 30 t axle load, the stress at D10 area was 135.8 kPa, far less than the peak dynamic stress on the subgrade surface. Besides, as depicted from 150 groups of test results, under all axle loads, the peak dynamic stresses were found in the area under the sleeper.

Figure 6. Stress nephograms on the subgrade surface under 30 t train load.

4.2 Normality test on peak dynamic stress and stress at D10 area

For 150 groups of test results, the peak dynamic stress and stress at D10 area on the subgrade surface under different axle loads are depicted in Figure 7.

Figure 7. Peak dynamic stress and stress at D10 area on the subgrade surface under different axle load.

It is generally considered that the wheel-load is normally distributed (Chen et al. 2016). In this study, the Kolmogorov test is performed to verify the normal distribution of peak dynamic stress and stress at D10 area on the subgrade surface. The specific steps are as follows (Chen et al. 2016):

Assuming the distribution function of population X is $F(x)$, $F(x)$ is a continuous function but unknown, $(X_1, X_2, \dots, X_n)$ is a sample from the population, and $F_n(x)$ is a better estimate of $F(x)$. Since $F(x)$ is a nondecreasing function, the following test statistic is constructed:

$$D_n = \max_{0 < i \leq n} |F_n(x_i) - F_0(x_i)| \quad (3)$$

$$F_n(x_i) = i/n \quad (4)$$

$$F_0(x) = \Phi\left(\frac{x - \bar{x}}{s^*}\right) \quad (5)$$

where $(x_1, x_2, \dots, x_n)$ are observation values of order statistics $(X_1, X_2, \dots, X_n)$, μ is the mean of the observation values, $\mu = \bar{x}$, and σ is the standard deviation, $\sigma = s^*$.

Corresponding to significance level α , there exists a critical value $D_{n,\alpha}$. When D_n is less than $D_{n,\alpha}$, $F_n(x) = F_0(x)$ can be regarded as true, that is, at the significance level α , the population obeys a normal distribution.

The normality analyses of peak dynamic stress and stress at D10 area on the subgrade surface under various axle loads are performed according to the above method. Table 2 lists the D_n values at various conditions.

Table 2. Statistical table of the normality test of the peak dynamic stress and stress at D10 area on the subgrade surface.

Axle load (t)	D_n	$D_{n,\alpha}$		
	Peak dynamic stress	Stress at D10 area	$\alpha=0.05$	$\alpha=0.01$
25	0.048	0.033	0.058	0.069
27	0.028	0.035	0.058	0.069
30	0.035	0.042	0.058	0.069

It can be seen that D_n is less than $D_{n,\alpha}$ at significance levels α of 0.05 and 0.01. Therefore, the peak dynamic stress and stress at D10 area can be considered to obey the normal distribution. Taking the statistical results of 30 t axle load as an example, the histogram of the peak dynamic stress and stress at D10 area is shown in Figure 8.

Figure 8. Histogram of peak stress and stress at D10 area under 30 t axle load.

4.3 Maximum forecast of peak dynamic stress and stress at D10 area

According to the 3σ principle of normal distributions, the interval $(\mu-3\sigma, \mu+3\sigma)$ is regarded as the possible range for true values of the population. That is to say, $\mu+3\sigma$ is the maximum value of peak dynamic stress and stress at D10 area if they obey the normal distribution. Table 3 summarizes the inferred maximum value of the above two stresses under different axle load.

Table 3. Maximum peak dynamic stress and stress at D10 area on the subgrade surface.

Axle load (t)	Dynamic stress	Mean (kPa)	Standard deviation (kPa)	Maximum (kPa)
25	Peak dynamic stress	1418.35	150.61	1870.2
	Stress at D10 area	99.17	12.70	137.28
27	Peak dynamic stress	1648.94	190.17	2219.45
	Stress at D10 area	112.14	15.41	158.37
30	Peak dynamic stress	2021.79	313.64	2962.71
	Stress at D10 area	116.17	16.56	165.85

To verify the reliability of the probability analysis result, we compared the maximum stress at D10 area, the field test result and the calculation result obtained from the empirical formula (Zhang 2005), as shown in Table 4.

Table 4. Comparison of probability analysis result, test results and empirical calculations.

Analysis method	Thickness of ballasted track (cm)	Axle weight (t)	Speed (km/h)	Dynamic stress (kPa)
Field test result in Shuo-Huang railway	≥50	25	<100	93~110
		30	<100	100~123
Maximum stress at D10 area	35	25	120	137
		30	120	166
Empirical calculations result	35	25	120	104
		30	120	125

It can be seen that the maximum value of stress at D10 area inferred from the coupling model results is higher than the test value and empirical calculation value. The main reasons are as follows: (1) In the coupling model calculation, the stress concentration caused by the irregular shapes of ballast particles is considered, while the stress distribution is relatively uniform due to the stiffness of the earth pressure cell; (2) The thickness of ballasted track in the numerical model is smaller than that in the field test, which leads to the different degree of stress diffusion; (3) Particles in the numerical model simulate the clean ballasts with more obvious edges and angularities characteristics, whereas the ballast on the operating railway is fouled and worn, leading to the reduction of the contact stress.

5 CONCLUSION

In this paper, a novel shape reconstruction method was adopted to randomly regenerate 150 groups of ballast samples conforming to the statistical regularity of overall morphological characteristics of natural ballast. Based on it, a ballasted track-subgrade coupling model was established to investigate the stress distribution on the subgrade surface under three axle loads. The main conclusions are as follows:

The shape reconstruction method based on the probability density function of morphological indices can ensure that the modeled ballast has the same probability distribution as the real ballast. In addition, increasing the number of particles does not lead to the difference in the probability distribution.

The random formation of the main force chain inside the ballasted track and the discrete contact at the interface caused by the irregular shape of the particles are the major reasons for the random distribution of the dynamic stress on the subgrade surface. The peak dynamic stress was much higher than the stress at D10 area. No matter what the axle load was, the peak dynamic stress occurred under the sleepers.

Both the peak stress and stress at D10 area follow normal distributions. Due to the influence of factors such as ballast fouling, filling thickness and stress homogenization by the pressure cell, the larger average stresses were obtained in numerical models compared with field testing data.

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