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THE MANY-FACETED NOTION OF SPACE

On the Hypothesis of Mapping and the Observation of Spatial Phenomena

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Prologue: Making an Ansatz

When commencing a mathematical proof by writing “*Ansatz*:...,” one stresses that one begins with an idea or a notion, an informed guess, if you will. In this case, it is an assumption about the general form that the solution to the problem at hand is going to take, for example, an exponential function with a yet-to-be determined amplitude and wavelength. If chosen appropriately, the *ansatz* provides, during the course of the calculation, the exact solution to the problem; in this case, it returns the missing parameters (and is thus verified). The internationally recognized use of the loanword *ansatz* (from *ansetzen* which could roughly be translated as not only *[to] begin or [to] ensue [a project]* but also *[to] estimate*) is particularly revealing. One need not have any prior knowledge of heuristics to understand that at its core, the *ansatz* is a well-informed hypothesis, made on the basis of what we know and understand of the physical system.

The question about hypotheses and the terminology of a method is at the center of this article. Guided by a comparison between the notion of space in mathematics and the concept of space in architecture, we ask: how can we describe the notion of space, and what is the scientific nature of that description? What does our hypothesis consist of in an architectural design process, and in what way is that hypothesis put to the test?

The Manifold Idea of Space: From Mathematical Space to Urban Spaces

The Notion of Space in Mathematics and Physics

The novel *Flatland* by Edwin Abbott is as determinedly rational as it is paradoxically poetic – it is not for nothing that it bears the subtitle “A Romance of Many Dimensions.” Political parody and societal allusions aside, it is a notable thought experiment on the sensations of physical space and dimensionality. In a dream, the Sphere, living in Spaceland, offers advice to an inhabitant of the two-dimensional Flatland:

That Point is a Being like ourselves, but confined to the non-dimensional Gulf. He is himself his own World, his own Universe; of any other than himself he can form no conception; he knows not Length, nor Breadth, nor Height, for he has had no experience of them.¹

The notion of space in mathematics and physics derives first and foremost from a concept of dimensionality that relates to the degrees of freedom f . The aforementioned Point existing in a zero-dimensional space has no degree of freedom whatsoever, $f = 0$. In one dimension, the motion along a line is possible, $f = 1$, whereas in a two-dimensional space the Point could move along two axes, $f = 2$, and so forth.²

René Descartes' hypothesis about the nature of space adhered closely to this mathematical principle of dimensionality. Space was imagined as the abstract form of extensions, a set of placeholders, if you will, created by bodies that extend in three dimensions. Each point in space was to be defined by a set of coordinates, still known as Cartesian coordinates. It was a radical thought at the time, as it allowed to identify a point in space, not in terms of distance and angle (d, α) in relation to the observer – at that time a common concept in navigation – but, in absolute terms, as a point (x, y, z) in space. The idea led to the development of the concept of an absolute space by Isaac Newton, a theory that was then challenged by Gottfried Wilhelm Leibniz. Leibniz argued that space was purely relational, a culmination of relative locations of bodies.³ Their ideas of space coincided with separate notions of time and motion, that were, respectively, either understood as absolute entities or defined only by their relational character.⁴

With the theory of special and general relativity, Albert Einstein advocated the notion of inertial reference systems for which all laws of physics must be the same, *invariant*, and he introduced the concept of curved space-time (combining the three dimensions of space with the one dimension of time). To this day, the “problem” of space and time is not conclusively solved. In particular, the implications of the theory of general relativity, with its suggestion of a curved, continuous space, on the one hand, and the contrasting idea of quantized fields, a notion that stems from quantum physics, on the other, have raised a plethora of stimulating questions.⁵

The Notion of Space in Architecture

Remarkably, the reflection on the concept of motion, that is, the degrees of freedom, and the (conceivable) path of a particle in space and time have been key to finding a theory of space itself. In similar fashion, our understanding of urban space is intertwined with a contemplation of time and action. Seeking the distinction of urban spaces from mathematical space, Otto Friedrich Bollnow highlights the importance of a coordinate origin in the inhabited space, which relates to the idea of the viewpoint of the observer, who becomes the center point of the system.⁶ A similar thought is shared by Georges Perec:

We use our eyes for seeing. Our field of vision reveals a limited space, something vaguely circular, which ends very quickly to left and right, and doesn't extend very far up or down. If we squint, we can manage to see the end of our nose; if we raise our eyes, we can see there's an up, if we lower them, we can see there's a down.⁷

In short, the perception of space in architecture is subject to the (physical) observable just as much as the observer. The *red-blue plan* discussed in other chapters of this publication emphasizes the dialectics of architecturally delimited interior “warm” and exterior “cold” spaces. These are distinguished based, primarily, on the proportion of the surrounding buildings, streets, and fields,⁸ but they also implicitly depend on the insights and outlooks, perspective and vista, context and scale,⁹ and, one might add, are subject to the individual impressions of the observer.

The differentiation or autonomy of the theory of architectonic space from the idea of a physical space has not always been plainly stated. A sentiment famously expressed by Martin Heidegger is:

We have to recognize that the things themselves are the places, and not just belong in a place. In this case we would for a long time be forced to acknowledge a disconcerting fact: The place is not situated in the given space in the sense of a physical-technical space. The latter only unfolds as a result of the prevailing of places in an area.¹⁰

More optimistically, the manifold concepts of space, room, and place in architecture, rather than substituting or competing with physical models, add, figuratively, another dimension to our understanding of space.

On the Description and Classification of Spaces

Whether mathematical, physical, sociological, or urban sciences, all seek to provide a *description* of space that best matches reality and concisely directs our gaze to the aspects relevant to their respective fields (and, just as importantly, sets the stage for further investigations).¹¹

Finding a System and a Language

Descriptions in mathematics rely on statements that can be either true or false – there is no third option. From a set of axioms, theorems are logically derived and proven, and these statements are succinct and precise. As an illustrative example, the infinite set of natural numbers $N = \{1, 2, 3, \dots\}$ is identified using an axiomatic description, the four Peano axioms: first, 1 is a natural number. Second, every natural number n has a successor $n' = n + 1$ that is also a natural number. Third, 1 is not a successor of any natural number. Fourth, every natural number has only a single successor, in short, if $n' = m'$, it follows that $n = m$. These provide a simple and acute description of N that, subsequently, leads to more complex theorems.¹² In an analogous way, in physics, instead of deriving theorems from axioms we develop natural laws from *first principles*, that is, from postulated assumptions.¹³

In contrast, the description of space in architecture and urban design relies on a classification of (dialectic) manifestations of space, that is, a definition of a constituting taxonomy. The choice of categories or types makes or breaks the success of the mapping method. The observables must be selected with validity (and relevance) and should be well-defined (and confined). Innately, natural scientists relate the quantitative character of empirical laws to the scientific integrity of their subject. While beneficial, quantification is not inherently necessary; instead, the aim is a higher systematicity on the level of descriptions, explanations, and contextualization, for example.¹⁴ Put simply, the hypothesis that is scientific derives from more than the sum of its constituting facts:

The scholar should organise and arrange; one constructs science from facts, as one constructs a house from bricks; but a pile of facts is as much a science as a pile of bricks is a house.¹⁵

In many ways, finding a system corresponds to finding a suitable language. In the case of the physical space, mathematics is the language with which the relevant phenomena are articulated. In a description of architectonic and urban spaces, meanwhile, one defines a taxonomy, a vocabulary of classifying terms, parameters and variables, categories and sub-categories, and synonyms and antonyms, which set the stage for an analysis of the spatiality of the city. Depending on the focus, the language may differ, but it may also be more or less accurate in its expression of the reality or varyingly precise.

On the Observation of Phenomena

Clearly, the mapping of urban spaces is not defined by or limited to an objectifiable sentential logic. The perception of architectonic spaces is a complex result of quantifiable and non-quantifiable parameters sometimes subject to the perception of the individual.¹⁶ From this complexity there follows a certain lack of conceptual clarity. Sociocultural aspects can be quantified using statistical data; other qualitative aspects can to a certain extent be quantified by finding universal definitions. However, there is an arbitrariness to these definitions that cannot entirely be eliminated. The pivotal question is: is the variable we define *valid* in that it describes the substantial manifestations of space?

The private detective at the center of Sir Arthur Conan Doyle's *Sherlock Holmes* stories regularly expresses his frustration with his companion Dr. Watson, "you *see*, but you do not *observe*." It is a fine yet obvious distinction that points the finger at a critical aspect of the mapping of spaces. In assigning values, classifying categories of space, we perform an analysis on our surroundings, which inherently implies an interpretation of its characteristics:

[In] fact the belief that we can start with pure observations alone, without anything in the nature of a theory, is absurd.... Observation is always selective. It needs a chosen object, a task, an interest, a point of view, a problem. And its description presupposes a descriptive language, with property words; it presupposes similarity and classification, which in their turn presuppose interests, points of view, and problems.¹⁷

The hypothesis we make consists in the intrinsic statement about the validity and demarcation of the selected variables. We do not merely *see* (unbiased and without judgment), but we select, interpret, and *observe*. In the case of the red-blue plan, the classification of spaces relies on an observation of *phenomena*. By definition, it draws our attention to certain aspects of our perception of space, in the context of a plethora of unknown parameters, complex relationships, and correlations.

On the Hypothesis and Its Falsification

By (in fact, mathematical) definition, a mapping consists in a functional relationship. In this case, the mapping assigns a term (e.g., warm or cold, red or blue) dependent on the practically observable parameters (e.g., proportion of the street's section). Its empirical, or

phenomenological nature, lies in the fact that it relies on practical observations to form a hypothesis about the spatiality of the city.¹⁸ The hypothesis and the observation go hand in hand, and, in an almost iterative process, the one constantly enhances the other:

The problem “Which comes first, the hypothesis (H) or the observation (O)?” is soluble; as is the problem, “Which comes first, the hen (H) or the egg (O)?” The reply to the latter is, “An earlier kind of egg”; to the former, “An earlier kind of hypothesis.”¹⁹

According to Karl Popper, “the criterion of the scientific status of a theory is its falsifiability, or refutability, or testability.”²⁰ However, due to its elements of subjectivity, the mapping lacks in reproducibility, as there are cases of transition where the subtle distinction between two opposites of spatial manifestations can neither be made unambiguously, nor be proven conclusively. Is it possible, still, in accordance with Popper’s *conjectures and refutations*, to *falsify* a mapping?

The answer is that the theory of *trial and error*, while not applicable to evaluate the truth in the single instance of spatial phenomena (for how could one prove or disprove a definition) might on a higher level be applied to the *method*. By critical revision of the taxonomy, by determining knowledge gaps and improving definitions where they lack in precision or completeness systematicity is increased.

Alternatively, as once suggested by Oswald Mathias Ungers²¹ and contemplated by Jasper Cepl,²² the idea, or topic (*Thema*) of an architectural intervention (resulting from the analysis) may itself constitute the actual hypothesis, which is then subject to confirmation or rebuttal.

The architecture can neither describe nor depict; it has to set itself a theme, it is not provided freely. And this is the engagement, namely the assignment of the topic. And that is a difficult feat, and that is what it is all about. One can make it entirely about this [assignment], by, in the spirit of Karl Popper, equating the idea with the hypothesis. As architect, I see my role in the establishment of ideas and the statement of hypotheses, which might then be refuted, or might not be upheld, as all other factors, of economics, usage etc. arrive on the scene.²³

In both cases, it is not the variable that is falsified, but rather the hypothesis that is formed on the manifestations of architectonic space in general. A mapping allows the depiction of a chosen set of spatial characteristics and can reflect their evolution in time. It augments our understanding about the prevalent context and the consequences of an architectural intervention.

The falsifiable hypothesis then lies in the choice of classification itself. Instead of asking, “Does *this* space really show an interior spatiality?” we might enquire, “Can I *discern* the two qualities of inner and outer spaces in my observations?” Alternatively, “Does the intervention provide the appropriate *idea*, and *how* can I describe its underlying qualities or that idea more systematically?” As previously noted by Cepl,²⁴ whether this is in Popper’s spirit remains to be discussed.

Epilogue: The Objective of Mapping the Medium-sized European City

We use the mapping of spaces as an instrument in the designing process. It is a *Versuch*, an experiment, an effort, a trial. Its result will not be limited to the Boolean return values “true”

or “false.” Rather, the experiment that we conduct is the testing of the validity of the defined language. A critical discussion of our taxonomy enhances our observations and our understanding of the spatiality of the city. A phrase attributed to Johann Wolfgang Goethe is:

The people are put off by the fact that the true things are so simple; they should bear in mind that they will find it hard enough to put it to practical use.²⁵

If viewed as a method of designing, then, above all, the mapping of urban spaces is judged both against the insights gained from the analysis and by its implications for the architectural intervention that finds either acknowledgment or rebuttal. The objective of a mapping of spaces is nothing less than to “decipher a bit of the town.”²⁶ In this broad sense, we make an *ansatz*, an informed assumption about a suitable classification that depicts an architectural dimension of space, heightening our awareness of the spatiality of the city.

Notes

- 1 Edwin Abbott, *Flatland: A Romance of Many Dimensions*. London: Seeley & Co. Reprinted 1998. London: Penguin Books.
- 2 We assume here a single idealized point, reduced to its center of mass. For a system of mass points, there are additional rotational degrees of freedom, again depending on the dimensionality of the space.
- 3 The disagreement between Leibniz and Newton culminated in the well-known *Briefwechsel* (exchange of letters) between Clarke, who was a devoted student of Newton’s, and Leibniz. It discusses their diverging ideas on many other grand topics of physics and philosophy, including the subjects of matter, atomic particles, reason, freedom, and God.
- 4 Not unambiguously, the terms relational and absolute space are sometimes used to describe architectural spaces; most prominently they appear in theories formulated by Henri Lefebvre and Martina Löw, alluding to the idea that space is induced by constructing relations (of abstract ideas, or objects, respectively).
- 5 Unifying concepts such as the theory of loop quantum gravity, for example, suggest that space is made up of quanta of gravity, which, needless to say, has dramatic consequences for our understanding of space, matter, and, indeed, time.
- 6 Bollnow, O. F. 1960. Der erlebte Raum. In *Universitas* 15. H.4. 397–412. As reprinted in: *Architektur. Raum. Theorie. Eine kommentierte Anthologie*, eds. A. Denk, U. Schröder, and R. Schützeichel. Tübingen-Berlin: Ernst Wasmuth Verlag, 2016, pp. 456–470.
- 7 Perec, G. 1974. *Espèces d’Espaces*. Paris: Editions Galilée. English Edition: 2008. *Species of Spaces and Other Pieces*. London: Penguin Group.
- 8 See Schröder, U. 2005. Pardié. *Konzept für eine Stadt nach dem Zeitregime der Moderne*. Köln: Verlag der Buchhandlung Walther König.
- 9 On the relevance of proportion and scale, see, for example, Tschanz, M. 2018. Proportion! Und Maßstab! In: *Proportionen und Wahrnehmung in Architektur und Städtebau*, eds. A. Gerber, T. Joanelly, and O. A. Franck. pp. 35–45. Berlin: Reimer Verlag.
- 10 Translation by the author. The original quotation reads:

Wir müssen erkennen, daß die Dinge selbst die Orte sind und nicht nur an einen Ort gehören. In diesem Falle wären wir auf lange Zeit hinaus genötigt, einen befremdenden Sachverhalt hinzunehmen: Der Ort befindet sich nicht im vorgegebenen Raum nach der Art des physikalisch-technischen Raumes. Dieser entfaltet sich erst aus dem Walten von Orten einer Gegend.

In: Heidegger, M. 1969. *Die Kunst und der Raum / L’art et l’espace*. St. Gallen: Erker-Verlag

- 11 In physics, the motivation is a unifying theory or law that explains reality as well as predicts hitherto unknown facts; in architecture, the objective is a (hermeneutic) understanding of the spatiality of the city as a starting point for a hypothesis of an architectural intervention.
- 12 Remarkably, the second axiom for natural numbers leads to a principle called complete induction, which is highly useful in the proving of quite a number of theorems.

- 13 See Einstein, A. 1953. *Mein Weltbild*, ed. C. Seelig. Zürich: Europa Verlag AG.
- 14 The point about quantifiability and systematicity and its dimensions is more adeptly made by Hoyningen-Huene, P. 2008. "Systematicity: The Nature of Science." *Philosophia*, 36(2): 167–180. <https://link.springer.com/article/10.1007/s11406-007-9100-x>
- 15 Translation by the author, from the German translation *Der Gelehrte soll anordnen; man stellt die Wissenschaft aus Tatsachen her, wie man ein Haus aus Steinen baut; aber eine Anhäufung von Tatsachen ist so wenig eine Wissenschaft, wie ein Steinhäufen ein Haus ist.* In: Poincaré, H. 1914. *Wissenschaft und Hypothese*. Deutsch von F. und L. Lindemann. Leipzig: B.G. Teubner.
- 16 The philosophical question of truth, knowledge, and cognizance is an altogether different matter which we cannot address in the context of this brief chapter.
- 17 Popper, K. R. 1963. *Conjectures and Refutations. The Growth of Scientific Knowledge*. London: Routledge & Kegan Paul plc, reprinted 1991 by Routledge.
- 18 Still, the mapping is also not merely a collection, *Sammlung*, of phenomena, but rather the descriptive representation of urban spatial manifestations (and the one phenomenon of interior or exterior spatiality, for example).
- 19 See Popper 1963.
- 20 Ibid.
- 21 Pehnt, W., G. Behnisch, and O. M. Ungers. 1981. Den Ort suchen, den Ort setzen. In: *Bauwelt*, 19: 774–779. <https://www.bauwelt.de/dl/744117/1981-19-0774-0779.pdf>
- 22 See Cepl, J. 2018. Architectural Design in the Interplay between Conjectures and Refutations: Karl Popper and the Architects. In: *Architettura e saperi | Architecture and Knowledge*, eds. S. Hildebrand, D. Mondini, and R. Grignolo, with B. Pedretti. Mendrisio: Mendrisio Academy Press, pp. 207–215.
- 23 Translation by the author, original wording:

[...] die Architektur kann weder beschreiben noch abbilden, sie muß sich das Thema setzen, sie kriegt es nicht umsonst geliefert. Und das ist die Auseinandersetzung, die Setzung des Themas. Und die ist schwierig, und darum geht es eigentlich bei der Sache. Sie können sie auch damit bezeichnen, indem Sie wie Karl Popper beispielsweise die Idee gleichsetzen mit einer Hypothese. Ich sehe meine Aufgabe als Architekt darin, Ideen zu setzen und Hypothesen aufzustellen, die dann widerlegt werden können, die unter Umständen auch als Idee sich nicht durchhalten lassen, wenn all die anderen Faktoren der Wirtschaft, der Benutzung usw. dazukommen

(See Pehnt 1981)
- 24 See note 22.
- 25 English translation by the author, the original quotation reads as follows: "Die Menschen verdrießt's daß das Wahre so einfach ist sie sollten bedenken, daß sie noch Mühe genug haben, es praktisch zu ihrem Nutzen anzuwenden." In: Goethe, J. W. 1907. *Maximen und Reflexionen. Aphorismen und Aufzeichnungen. Nach den Handschriften des Goethe- und Schiller-Archivs*, ed. M. Hecker, Weimar: Verlag der Goethe-Gesellschaft.
- 26 See Perec 1974.

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